

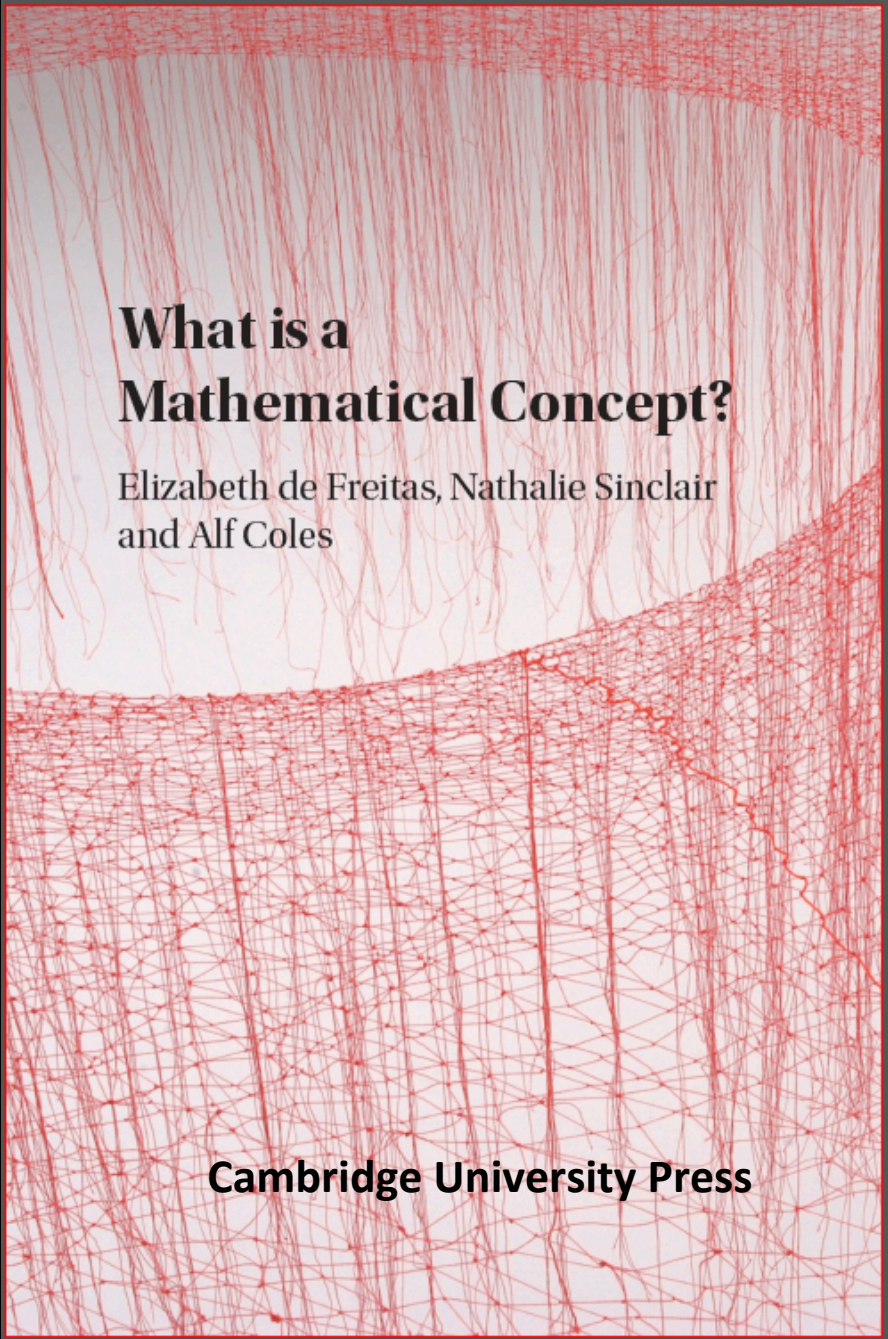
What is a mathematical concept?



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**What is a
Mathematical Concept?**

Elizabeth de Freitas, Nathalie Sinclair
and Alf Coles

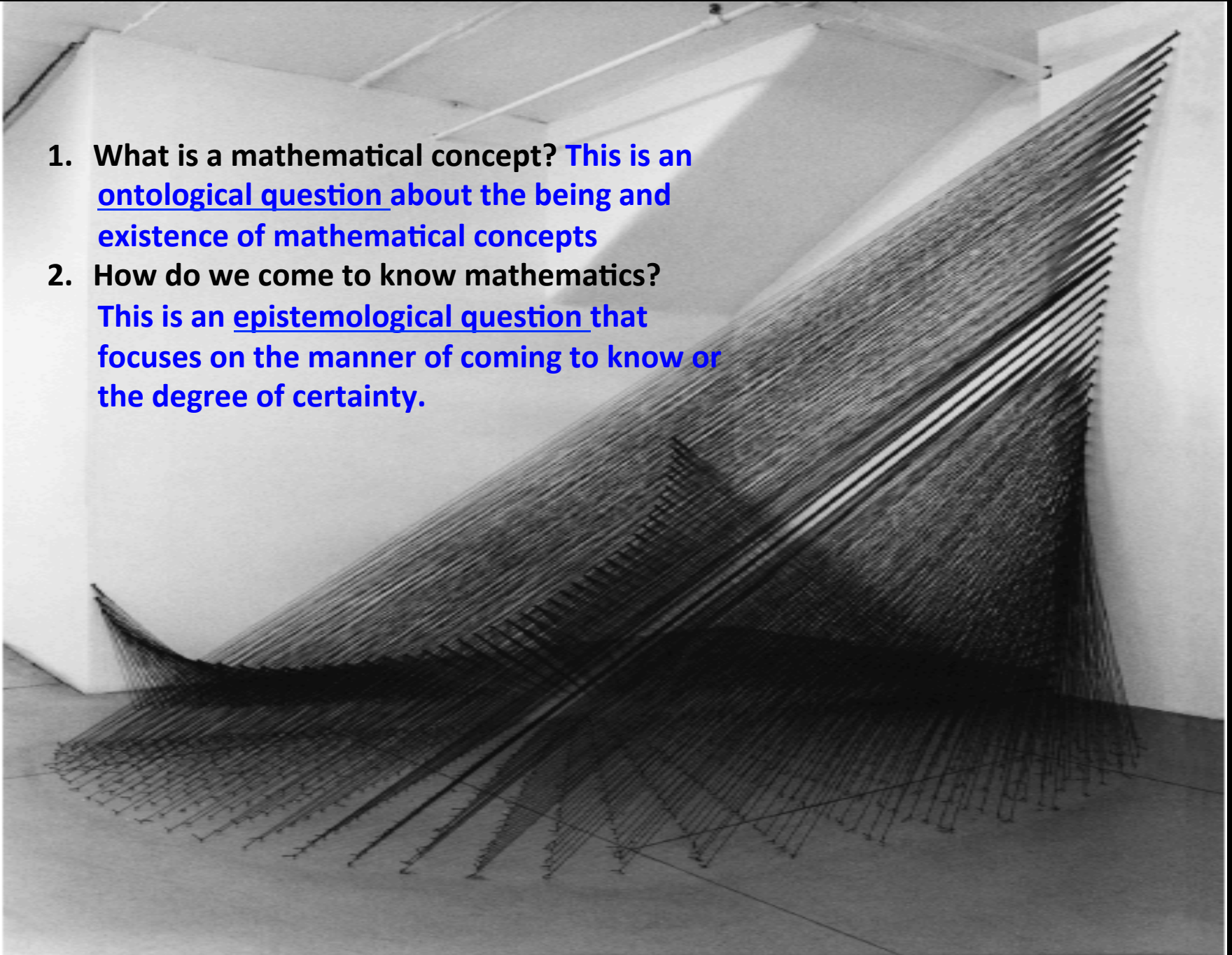
Cambridge University Press

Using the Philosophy of Mathematics in Teaching Undergraduate Mathematics



Mathematical
Association of
America Press
(2017)
B. Gold, C.
Behrens, R.
Simons (Eds.)

1. What is a mathematical concept? **This is an ontological question about the being and existence of mathematical concepts**
2. How do we come to know mathematics? **This is an epistemological question that focuses on the manner of coming to know or the degree of certainty.**



Opening up the question

1. When does a mathematical concept become a mathematical concept?
2. What is the relationship between mathematical concepts, discourse and the material world?
3. How do our theories of cognition and learning convey particular assumptions about the nature of mathematical concepts?
4. What is the role of diagrams, symbols and gestures in making mathematical concepts?
5. Are particular mathematical concepts productive of particular ways of being together? What are the political implications for particular concepts?

A philosophical tradition

- Do they exist in the mind or outside the mind?
- Are they **corporeal** (embodied in material objects) or **incorporeal** (ideals that exist independent of the human mind)?
- Are they bound to that which is perceptible, or can they continue to exist independent of their material form?

Conventional Images of Mathematics Ontology

- **The *cognitivist (mental entities)*** (Descartes (1596-1650), Kant (1724-1804) claims that they exist in the mind and that they are created by the mind. Some claim that we create these universals based on sense perception and some say they are innate and do not require perceptual stimulation.
- **The *realist*** (Plato (428 BC), Frege (1848-1925) , Gödel (1906-1978) claims that universals exist outside the mind and are independent of all human thought.
- **The *nominalist*** (Hilbert (1862-1943)) claims that they do not exist outside of language. Some claim that the words and symbols we use are mere shorthand for longer ways of expressing the same idea and some claim that statements with such terms are simply untrue in the sense that they refer to nothing.

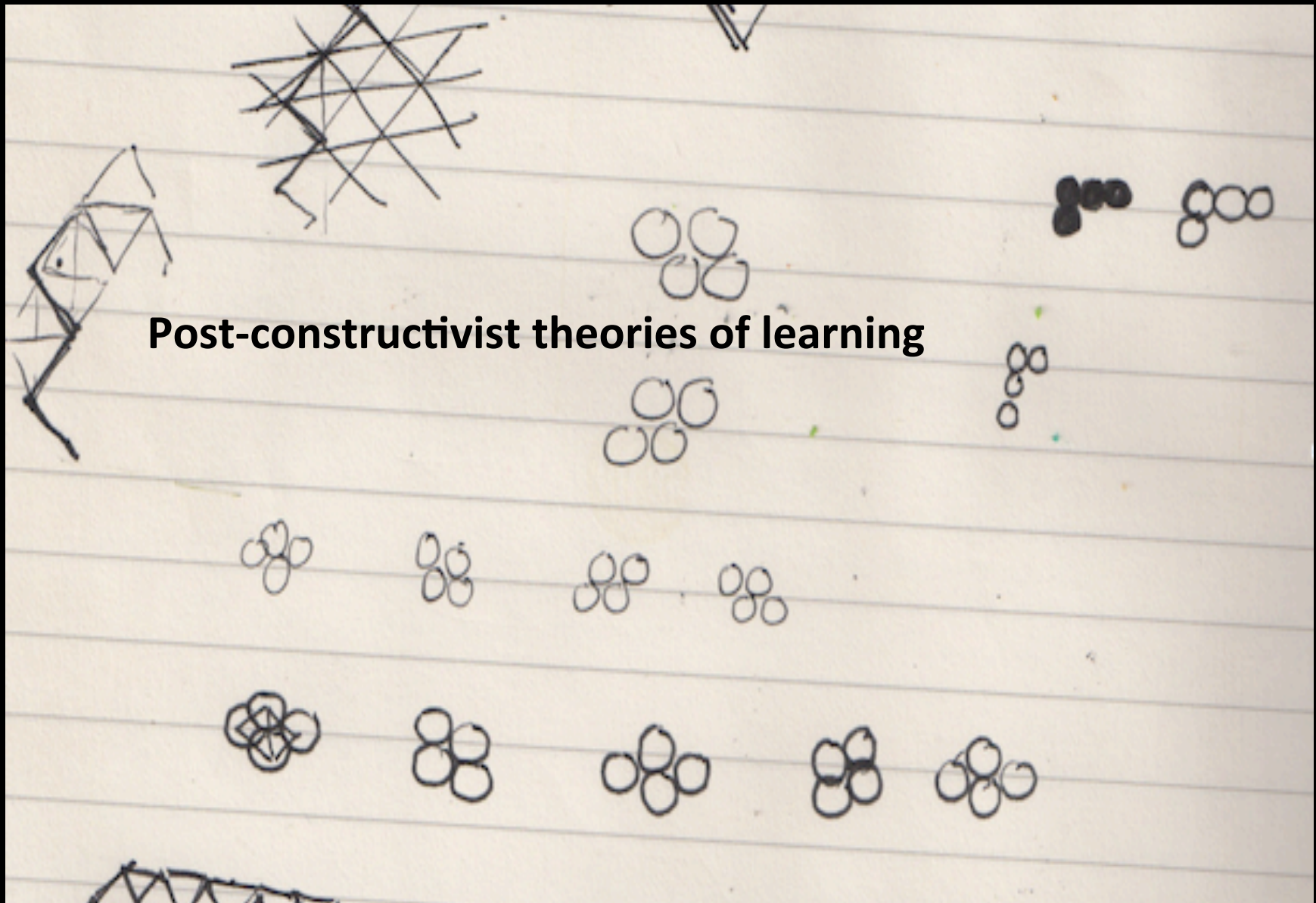
The legacy of the Kantian mental schema

- Constructivist theories of learning
- Piaget (1896-1990)
- Vygotsky (1896-1934)
- “A concept emerges and takes shape in the course of a complex operation aimed at the solution of some problem” “A concept is not an isolated, ossified, and changeless formation” (Vygotsky, 1934, p. 54)
- Anna Sfard, Michael Roth

Constructivism continues to rest on a dualist ontology

- Emphasis on **cognitive images** or 'mental concepts' ascribes to the human mind a **consciousness or intuition or faculty** that is capable of bringing together the ideal forms (**triangles, numbers, etc**) that are unchanging and eternal (**the realm of being or essence**) with the physical realm (**the realm of becoming or change**).
- There is a strong dualism (**between mind and body**) at work in this approach, and this dualism plays out in different pedagogies and curriculum.

Post-constructivist theories of learning



Brent Davis (2017)

- Concepts as **biological species**
- Concepts are “**memeplexes**”, with a life form and a networked living body that evolves in complex ways

Ricardo Nemirovsky (2017)

- Critiques the Aristotelian theory that concepts are like branches on a tree (polygon, quadrilateral, square, ...)
- **growth and decay**
- **Inhabiting** a concept

Gilles Chatelet (2000)

- The diagrammatic & gestural
- **Diagramming** is crucial for the making of new concepts

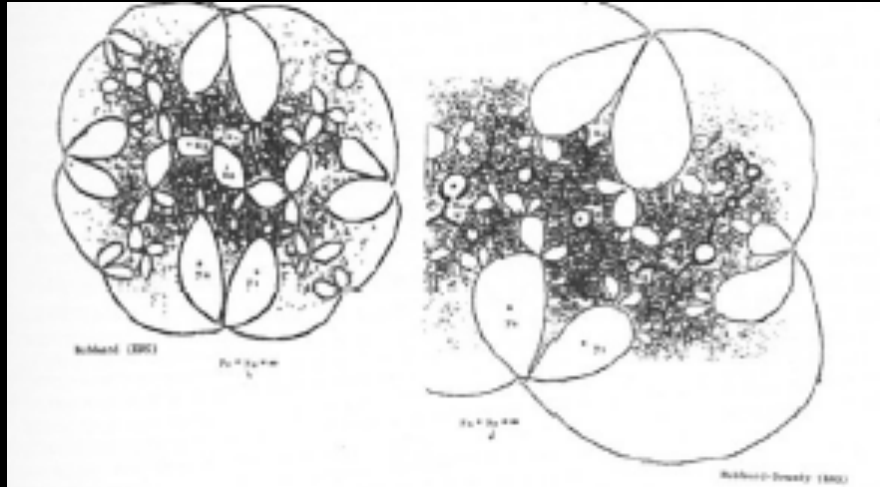
A problematics

- **Simon Duffy** (Gilles Deleuze): Problems operate as the engine of mathematical invention, such that the emergent “solutions” are clusters of concepts
- **Imre Lakatos** (1922-1974): **Falibilism**: Concept deformation, stretched and mutated

Problematics

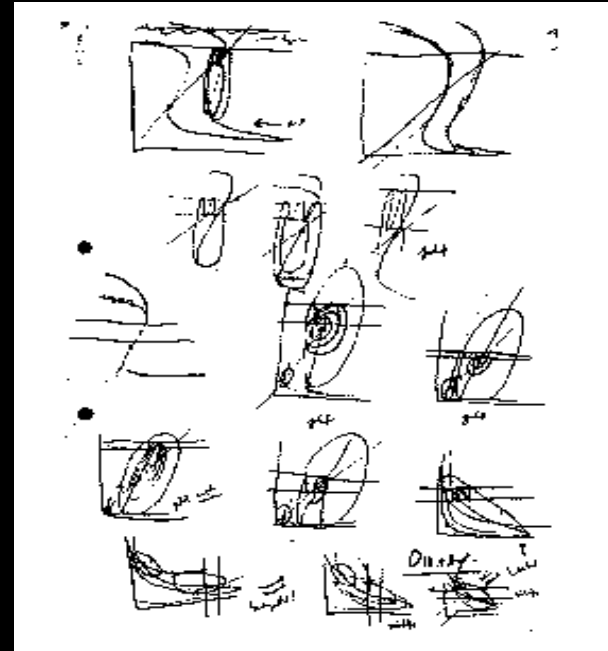
- **Arkady Plotnitsky (2017)** studies the work of Riemann. He states that a mathematical concept:
 1. emerges from the co-operative confrontation between mathematical thought and chaos;
 2. is multi- component;
 3. is related to or is a problem; and
 4. has a history

Proposition 1: Concepts Are Not Mental Constructs Abstracted from the Material World



Mating
Rabbits

Adrien Douady



Rössler Attractors

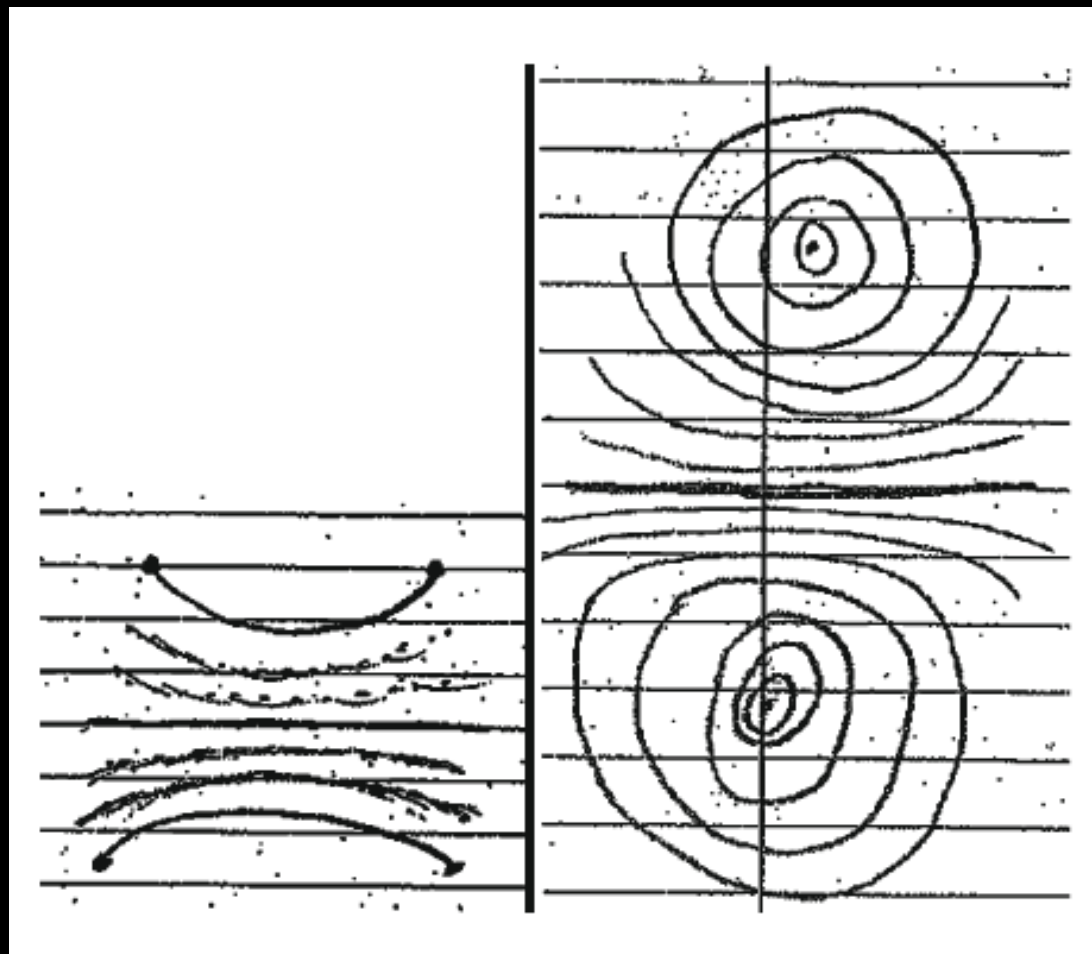
Otto E. Rössler

Proposition 2: Concepts Are Not Merely Metaphors or Representations



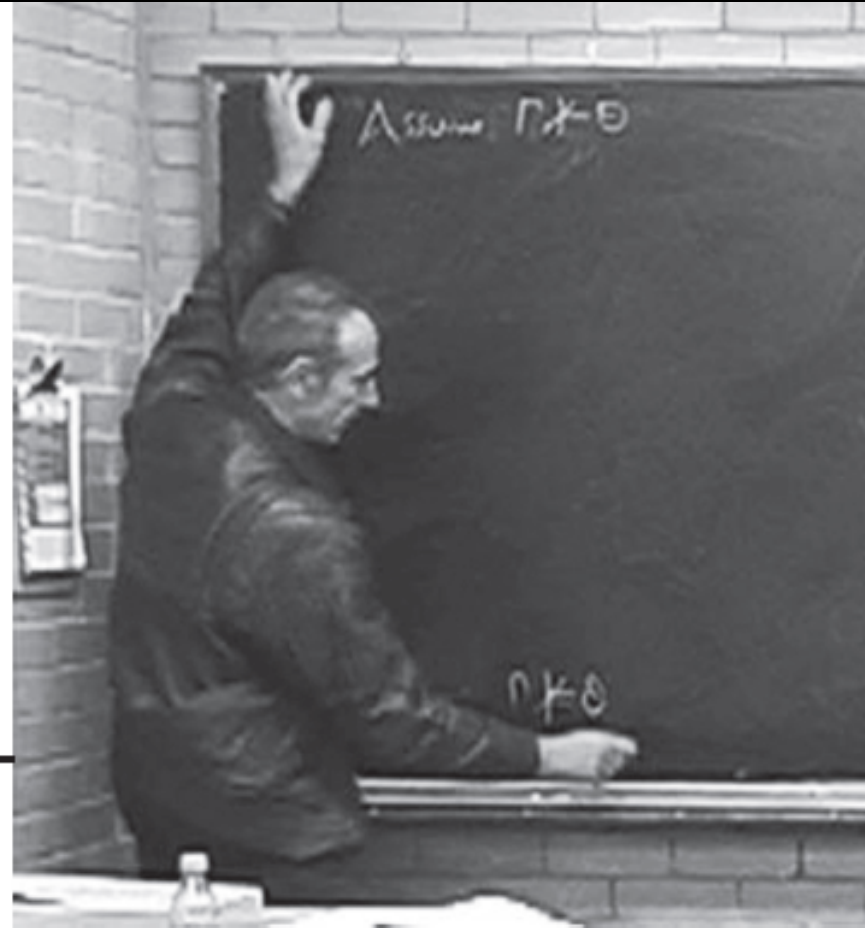
The two girls are analysing **the movement in terms of position**

**Proposition 3: Concepts Are Vibrant and Indeterminate, Having
One Foot in the Virtual and One in the Actual**

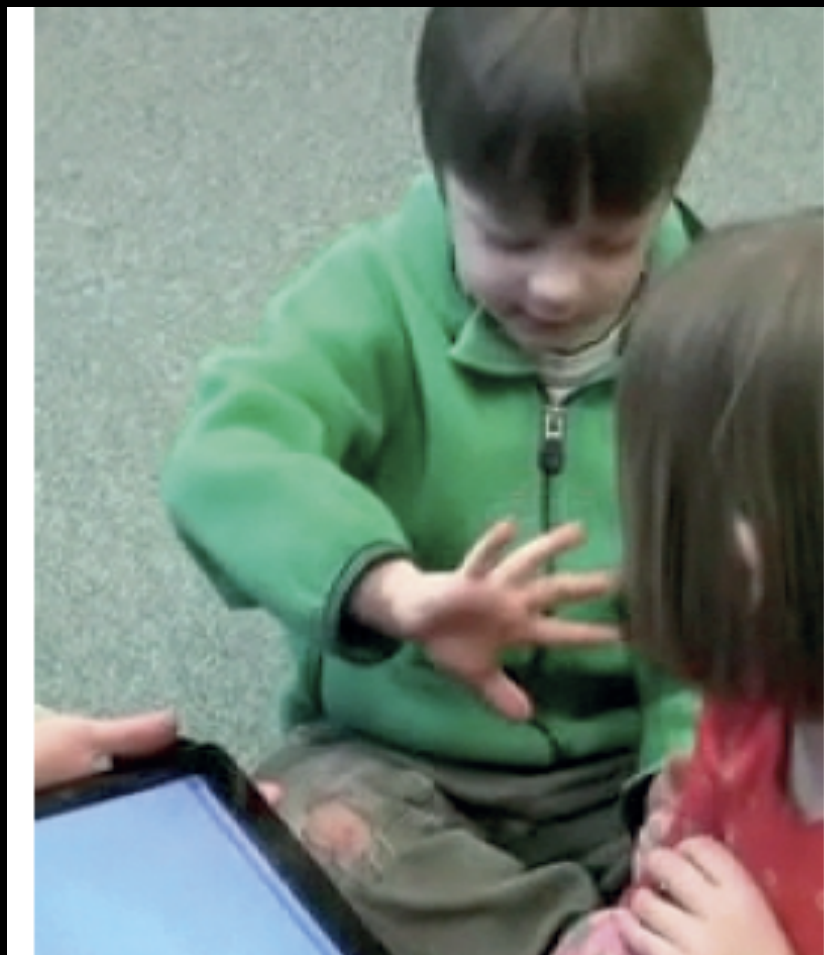


Proposition 4: Concepts Operate as Both Logical and Ontological Devices

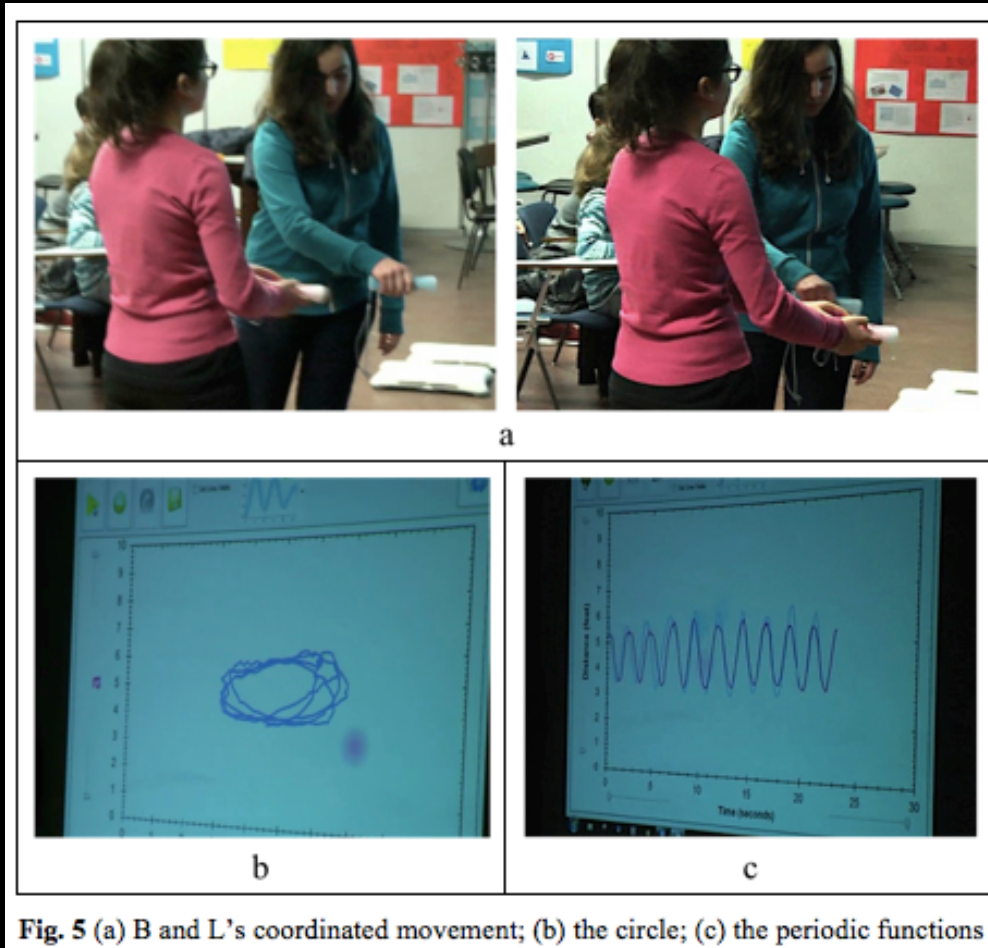
- 1 So. ((erases theorem))
- 2 ((starts writing at top))
- 3 What we've done here, is assumed.
- 4 It's not-, it's not possible to prove
- 5 theta from Gamma. And somewhere
- 6 at the bottom of the page.
- 7 ((starts writing at bottom))
- 8 We want to get down to showing:
- 9 Therefore Gamma does not logically
- 10 imply theta
- 11 ((finishes writing))



Proposition 5: There Is No a priori Logical Ordering between Mathematical Concepts



Proposition 6: Concepts Emerge from Aesthetico-ethical Acts



Conceptual learning

- Cutler and MacKenzie (2011) suggest that learning is that which sustains the mobility of concepts – in other words, ‘that which resists determination as knowledge’ (p. 68).
- The ontological force of concepts, not as transcendental ideals that shape reality, but as immanent forces at play in particular problems, as part of world-building or worlding processes, operating alongside the logical constraints of mathematical proof.



Thank you

