Teaching for Robust Understanding of Mathematics (TRU Math)[1]

Coding Scheme Version GG, 20130308 – Printable version

SUMMARY RUBRIC	Level	Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
	1	Classroom activities are purely rote, **OR** disconnected or unfocused, **OR** consequential mistakes are left unaddressed.	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
	2	The mathematics discussed is relatively clear and correct, **BUT** connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Students have a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
	3	The mathematics discussed is relatively clear and correct, **AND** connections between procedures, concepts and contexts (where appropriate) are addressed and explained.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

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			Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
W	*Whole Class Activities: Launch, Teacher Exposition, Whole Class Discussion*		*How accurate, coherent, and well justified is the mathematical content?*	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to meaningful participation for all students?*	*To what degree are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas (when potentially valuable) or address misunderstandings when they arise?*
	***Note**:* *On the score sheet, Circle one of **L** / **E** / **D** if the episode is primarily of that type.*	1	Classroom activities are purely rote, **OR** disconnected or unfocused, **OR** consequential mistakes are left unaddressed.	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
		2	The mathematics discussed is relatively clear and correct, **BUT** connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Students have a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
		3	The mathematics discussed is relatively clear and correct, **AND** connections between procedures, concepts and contexts (where appropriate) are addressed and explained.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's' ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

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G	*Small **G**roup work*		*How accurate, coherent, and well justified is the mathematical content?*	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support/ group dynamics provide access to meaningful participation for all students?*	*To what extent does the teacher support/ group dynamics provide access to "voice" for students?*	*To what degree does the teacher monitor and help students refine their thinking within small groups?*
	Note: *If students are engaged in early brainstorming, the role of the teacher is to support students in exploring and justifying. This is the reason for "ORs" in the scoring.*	1	Students' mathematical ideas are incoherent or incorrect **AND** they go unaddressed by the teacher.	Activities or teacher intervention constrain students to rote activities such as applying familiar, straightforward procedures.	Disengagement is not addressed.	Teacher interventions, if any, either constrain students to producing short responses to the teacher **OR** do not address clear imbalances in group discussions.	The teacher does not engage students in discussion, or actions are simply corrective.
		2	Discussion of mathematics is relatively clear and accurate, **BUT** students are not expected to or supported in justifying their ideas.	Activities offer possibilities of conceptual richness or problem solving challenge, **BUT:** students are either left unsupported when lost, **OR** the teacher's interventions remove the challenge.	All team members appear to be engaged, **OR** the teacher makes moves to engage team members who are not engaged.	At least one student has a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": students are not encouraged to build on each other's ideas.	Teacher points to student errors or responds to questions, but does not help students build on nascent ideas or encourage group discussion of potential problems.
		3	Explanation of and justification for the central mathematical ideas is accurate, **OR** teacher supports students in focusing on central mathematical ideas and explaining and justifying them.	Activities offer possibilities of conceptual richness or problem solving challenge, **AND** students are provided scaffolding (if they are lost) that enables them to grapple with the tasks at hand, but for which the challenge has not been removed.	Everyone in the team contributes meaningfully, **OR** teacher makes moves to try to give all team members access to making meaningful contributions.	At least one student puts forth and defends his/her ideas. The teacher may ascribe ownership for students' ideas in exposition, AND/OR students respond to and build on each other's ideas.	The teacher solicits student thinking, **AND** subsequent discussion responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

		Mathematical Focus, Coherence and Accuracy	**Cognitive Demand**	**Access**	**Agency: Authority and Accountability**	**Uses of Assessment**
P Student **P**resentations		*How accurate, coherent, and well justified is the mathematical content?*	*Sometimes coded - See notes*	*Sometimes coded - See notes*	*To what degree are students the source of presented ideas and response to presented ideas?*	*Sometimes coded - See notes*
			N/A, or…	N/A, or…		N/A, or…
***Notes**:* *• If the episode is in essence a conversation between student presenter(s) and teacher, then Cognitive Demand and Access, and Uses of Assessment (for the class) are coded as N/A.* *• If the presentation becomes a conversation that involves the whole class for 45 seconds or more, then all five dimensions are coded.*	1	Presenters' mathematical ideas are incoherent or incorrect **AND** inaccuracies and incoherence are not addressed.	Presentation and classroom discussion focus on familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	Presenter role is structured by teacher/text and student is narrowly constrained in response to teacher questions.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
	2	The mathematics presented is relatively clear and correct, BUT the presenters are not encouraged to or supported in justifying their ideas.	Presentation offers possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" these possibilities and focus on procedures.	The presentation evolves into whole class activity. There is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Presenters have the opportunity to demonstrate individual proficiency, without being tightly constrained by text or teacher. **BUT,** the discussions do not build on students' ideas. (*To qualify as an *idea*, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
	3	The mathematics presented is relatively clear and correct, **AND** either includes justifications or explanations **OR** the teacher encourages students to focus on central mathematical ideas and explaining and justifying them.	The teacher's hints or scaffolds support presenters and/or class in "productive struggle" in building understandings and engaging in mathematical practices.	The presentation evolves into whole class activity. The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.	Student presentations result in further discussion of relevant mathematics, **OR** students make meaningful reference to other students'/groups' ideas in their presentations. (*To qualify as an idea, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher solicits student thinking and responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

			Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
I	Individual work		*How accurate, coherent, and well justified is the mathematical content?*	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent is there equitable access to meaningful participation for all students?*	*To what degree are students the source of presented ideas and response to presented ideas?*	*To what degree does the teacher monitor and help students refine their thinking within small groups?*
			May be N/A if there are insufficient data; or…	N/A, or…	N/A, or…	N/A, or…	N/A, or…
	Note: *Student seat work is coded as N/A unless the teacher is actively circulating through the classroom and consulting with students on an ongoing basis.* *Note that with a stationary camera it is impossible to see individual student work. Hence, unless there is evidence from the conversation, one cannot discern student errors.*	1	Materials for student work are rote, **OR** disconnected or unfocused, **AND** there is no evidence of consequential mistakes being addressed.	Materials demand no more than applying familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	Teacher shows or tells students how to do the mathematics, possibly correcting student work. Student ideas are not elicited or built upon.	Teacher actions are limited to corrective feedback or encouragement.
		2	Materials for student work provide some affordances for coherent mathematics, but teacher support is minimal and does not exploit them.	Materials offer possibilities of conceptual richness or problem solving challenge, but teaching interventions tend to "scaffold away" the challenges.	Students appear to be working, but there are no clear mechanisms for students who want or need support or attention to receive it.	One-on-one interactions give students the opportunities to talk about their ideas.	Individual interactions provide opportunities for students to discuss their thinking and teacher responses address such thinking explicitly (not simply correcting incorrect student work).
		3	The teacher's interventions with individual students support a coherent and connected view of the mathematics.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	Teacher's and/or surrogates' attention is clearly and widely available for those students who want it.	A score of 3 is not coded *unless* the student has ample opportunity and agency to develop his/her idea interacting with the teacher, **OR** the teacher takes the student idea up for class discussion immediately after individual student work comes to an end.	The teacher solicits student thinking and subsequent conversation or classroom discussion responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Robustness Criteria for Contextual Algebraic Tasks (CATS) -- Content Specifics

			1	2	3
RC1	Reading and interpreting text, and understanding the contexts described in problem statements.		One or more terms in the problem are reworded and/or defined.	The context (problem scenario) is elaborated or discussed and an explicit attempt is made to ensure students understand it.	The teacher or students link the context (problem scenario) with algebraic concepts (e.g. rate of change, proportion, variable, expression).
			1	**2**	**3**
RC2	Identifying salient quantities in a problem and articulating relationships between them		Salient quantities are identified but the relationships between quantities are not discussed (e.g., What are the slope and y-intercept?, "We know Jose's speed, we need to find the distance he travelled.")	Salient quantities are identified and local relationships between quantities are discussed (e.g. at a particular point: "what is the cost of plan A for 10 hours? Of Plan B?")	General covariation of quantities is discussed (e.g." as time increases, distance stays the same"; "when x increases by 1, y increases by 2") or the relevant family of functions is identified.
			1	**2**	**3**
RC3A	Generating representations of relationships between quantities		Algebraic representation(s) is(are) generated by way of practice (e.g., writing equation for a line given two points) without attention to the relationship(s) between variables or why the representation is a good choice for the given situation.	Algebraic representation(s) is(are) purposefully generated with explicit attention to either the relationship between variables *or* why the representation is a good choice for the given situation.	Algebraic representation(s) is(are) purposefully generated with explicit attention to the relationship between variables *and* attention to why the representation is a good choice for the given situation (e.g., "let's make a graph so we can see all the possible solutions to the equation").
			1	**2**	**3**
RC3B	Interpreting and making connections between representations		Representations are interpreted locally or in part (e.g., relevant quantities identified but relationship between quantities is not exploited (e.g., "from the graph, when x is 4, y is what?"). There are no connections between multiple representations.	Important global features of representations are explicated to highlight the covariation between quantities (e.g., relating the 'steepness' of a graph to a rate of change, using the representation to identify the family of functions relating the quantities) *or* connections among multiple representations are explored (e.g. focus on parameters in an equations and how the parameters affect the features of the representations, affordances of different representations may be highlighted).	Important global features of representations are explicated to highlight covariation between quantities *and* connections among multiple representations are explored (e.g. focus on parameters in an equations and how the parameters affect the features of the representations); affordances of different representations may be highlighted.

			1	2	3
RC4A	Executing calculations and procedures with precision		Arithmetic calculations are executed accurately, and any errors are corrected.	Algebraic procedures (see list) are executed accurately, and any errors are corrected.	Calculations and/or algebraic procedures are executed correctly with explicit attention to accuracy, or mistakes are caught and instruction involves guiding students to self assess and correct their calculational/procedural errors.
			1	**2**	**3**
RC4B	Checking plausibility of results		The plausibility of a solution is passively checked (e.g. teacher poses the question, "does this answer make sense?)	The plausibility of a solution is actively checked without attending to context (e.g., checking that the answer makes sense with regard to a representation or calculation, but not with the context).	The plausibility of a solution is actively checked in relationship to the context (problem scenario) to make sense of the solution (i.e. to judge the meaning, utility, and reasonableness of the results; NCTM, 2000, p. 296)
			1	**2**	**3**
RC5A	Opportunities for Student Explanations		An open-ended question is posed for students without explicitly soliciting an explanation or justification.	An explanation is explicitly requested of students, but the nature of the explanation is not specific; does not necessarily require an algebraic justification (e.g. "why?", "can you explain that?")	An explanation is explictly requested that focuses on algebraic reasoning (e.g. an algebraic representation, the qualitative relationship between quantities, or the problem context).
			1	**2**	**3**
RC5B	Teacher instruction about Explanations		Teacher explicitly provides guidelines on what is needed *generally* for good explanations,	Teacher explicitly provides guidelines on what is *generally* needed for good explanations *and* models such behavior.	Teacher provides feedback on and/or opportunities for students to incorporate the feedback to revise *specific explanations*.
			1	**2**	**3**
RC5C	Student Explanations and Justifications		Student gives a short explanation that describes only procedures (whether algebraic or non-algebraic), OR the explanation is unclear.	Student describes procedures, supporting them by either referring to the problem context or the underlying mathematical concepts.	Student generates a clear algebraic explanation (e.g. draws on an algebraic representation, the qualitative relationship between quantities, or the problem context) that extends beyond explaining how to do a procedure.