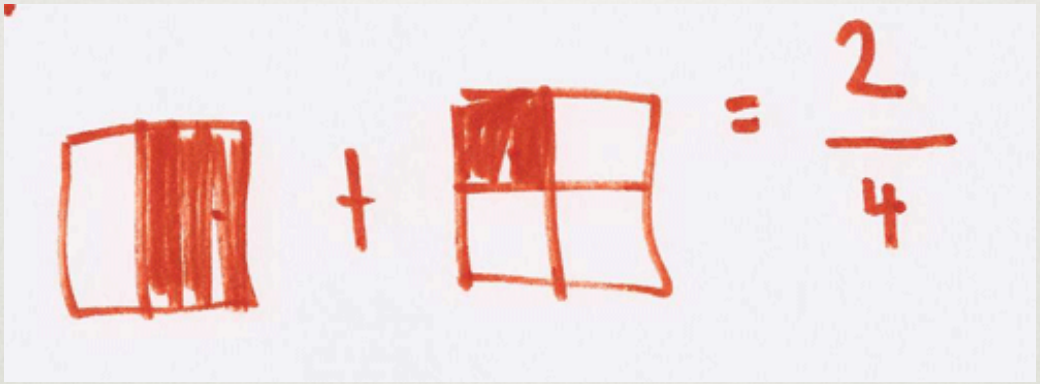
Disability Equity in Mathematics Education

Accessibility, Re-mediation, and Compensation

Katie Lewis
University of Washington

Michigan State University
January 25, 2023

$$\frac{1}{2} + \frac{1}{4} =$$



"there's two that are shaded and four that are left"

Mathematics Disabilities / Dyscalculia

persistent and pervasive learning difficulties specific to mathematics

Mathematical Disabilities / Dyscalculia

- Neurological difference in how students process quantity 3 
  - (Butterworth, 2010; Piazza et al., 2010)
- Similar to dyslexia – but for numerical information
- About 5-8% of students have a math learning disability (Shalev, 2007)
- Leads to significant difficulty with mathematics and errors.

Prior Research

mathematics disabilities / Dyscalculia

Prior Research On Math Dis.

- Behaviorist / Cognitive
- Cognitive **deficits**
- Problem located **within the individual**
- Remediations are about "remedying" (fixing) the student
- Outcome of Math disability —> Low achievement
- Low achievement is used as a proxy for Math Disability (Lewis & Fisher, 2016)

Issues

Deficit model is problematic

Pathologizes difference

Over-classification (Hanich, Jordan, Kaplan & Dick, 2001)

Anti-Deficit Perspective

- My positionality - White, queer, cisgendered woman, native English speaker, upper-middle class, with dyslexia

1. Disability Studies
2. Sociocultural perspectives - Vygotsky

Disability Studies

- Reject the deficit view of disability (medical model - focused on "curing")
- Social model
  - Acknowledges biological differences
  - Understands physical, sensory, and neurological differences as **natural and desirable** (Kafer, 2013)
  - Individuals are disabled not by biological differences, but by environments and societies that are not designed for them or don't value those differences (Barnes, 2003).
- **Disabled by contexts not by biological differences**

Vygotskian Theoretical Approach

- Typical Development (Vygotsky, 1978)
  - Biological and socio-cultural lines of development intersect
  - Human activity is mediated through tools/signs (language, numerals)
  - These tools/signs have been developed over the course of human history and are optimized for individuals with "typical" development

Vygotsky's

Fundamentals of defectology (1929/1993)

- For individuals with disability - Mediational tools are often **incompatible** with a individual's natural biological development —> **Inaccessibility**
    - Spoken language inaccessible for individuals who are Deaf
- Inaccessibility is addressed through creation of accessible meditational tools —> **Re-mediations**
    - Signed languages for the Deaf (e.g., ASL)
- If no accessible meditational tool is available - individuals with disabilities will often recruit alternative resources to accomplish same goals —> **Compensation**
    - Blind folks develop echolocation skills (Thaler, Arnott, & Goodale, 2011)

An alternative Approach

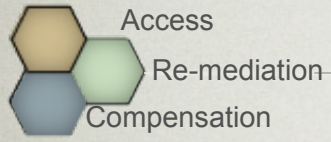
Prior Research On Math Dis.

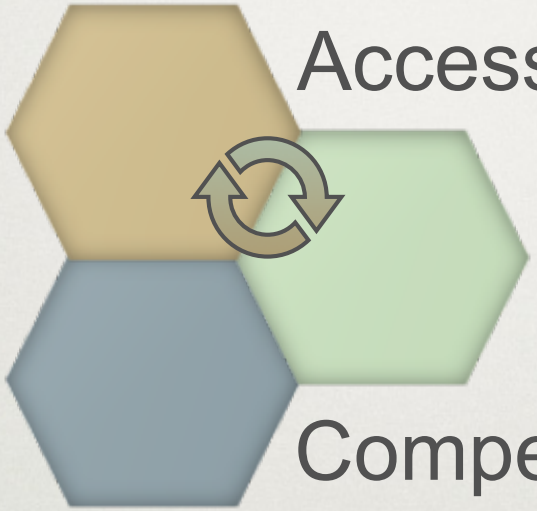
- Behaviorist
- Cognitive **deficits**
- Problem located **within the individual**
- Remediations are about "remedying" (fixing) the student
- Result of Math Disability —> Low achievement (failure)

My Alternative Approach

- Anti-deficit - (DS / Vygotsky)
- Cognitive **difference**
- Problem is an **issue of access** (external)
- Remediations -"**re-mediating**" fixing tools to provide access
- Result of Math disability —> **Compensation**

Overview of Talk





Access *(Lewis, 2014)*

(Lewis et al., 2020; 2022)

Re-mediation *(Lewis, 2017)*

Compensation *(Lewis & Lynn, 2018)*

Case Study #1 - Lisa

(Lewis, 2014)

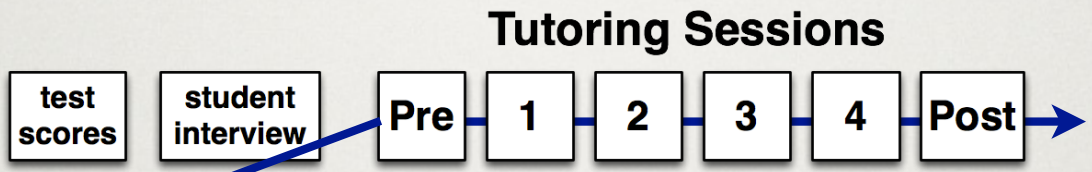
- Recruited "exceptional cases" - adults with significant difficulties with elementary math content (AND students who would meet criteria in other studies of math disability)

Identification Case Study
Student with math disabilities

Access

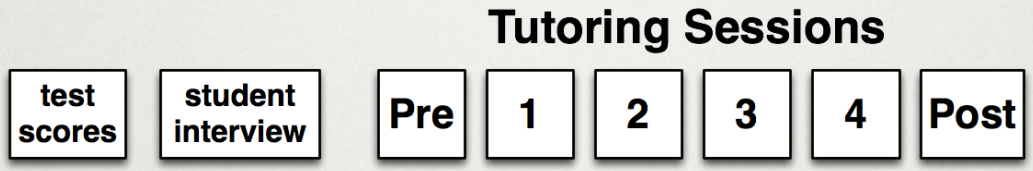
Students with possible math dis.
n=11

Fifth grade comparison students
n=5



Identification Case Study
Student with math disabilities

Students with possible math dis.
n=11



All benefited from the tutoring sessions.

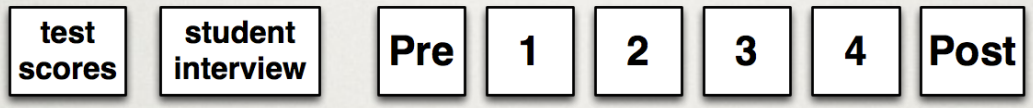
5th grade students:

- Average gain 15%
- Posttest scores above 60%

Identification Case Study
Student with math disabilities

Students with possible math dis.

n=11



Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction

All benefited from the tutoring sessions.

5th grade students:

- Average gain 15%
- Posttest scores above 60%

Identification Case Study
Student with math disabilities

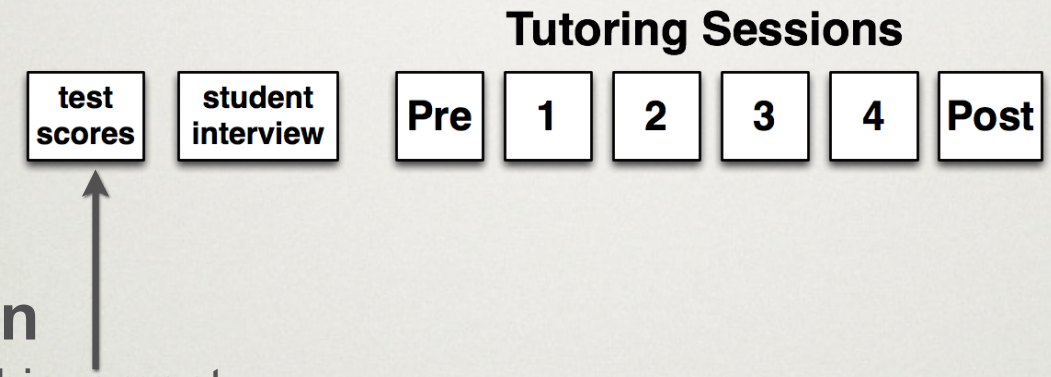
Access

Students with possible math dis.

n=11

Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction



All benefited from the tutoring sessions.

5th grade students:

- Average gain 15%
- Posttest scores above 60%

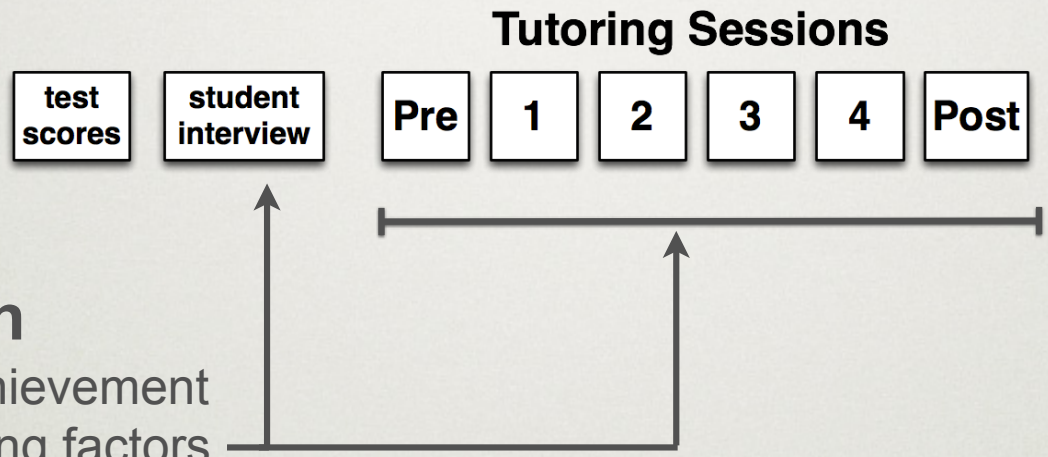
Identification Case Study
Student with math disabilities

Students with possible math dis.

n=11

Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction



All benefited from the tutoring sessions.

5th grade students:

- Average gain 15%
- Posttest scores above 60%

Identification Case Study
Student with math disabilities

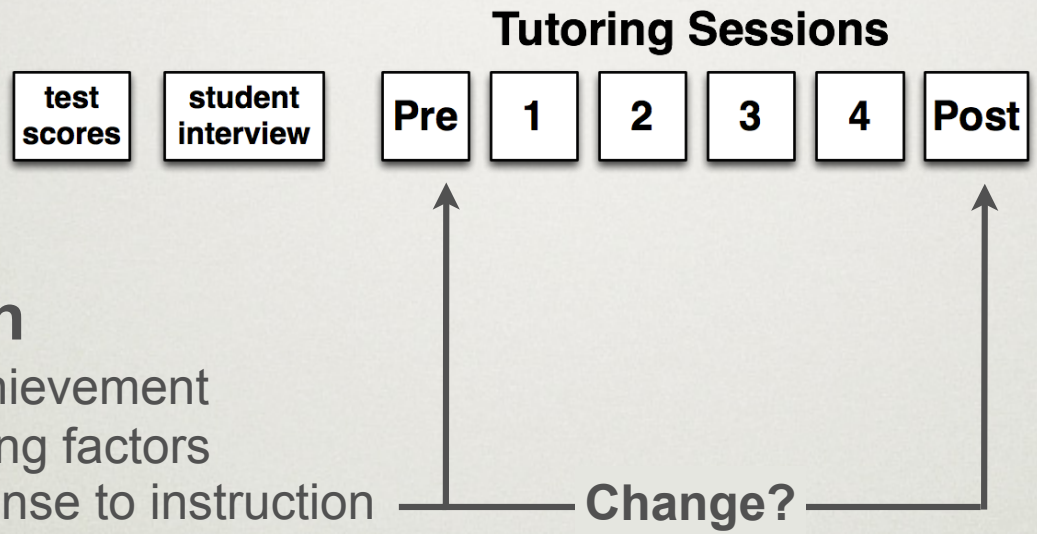
Access

Students with possible math dis.

n=11

Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction



All benefited from the tutoring sessions.

5th grade students:

- Average gain 15%
- Posttest scores above 60%

Identification Case Study
Student with math disabilities

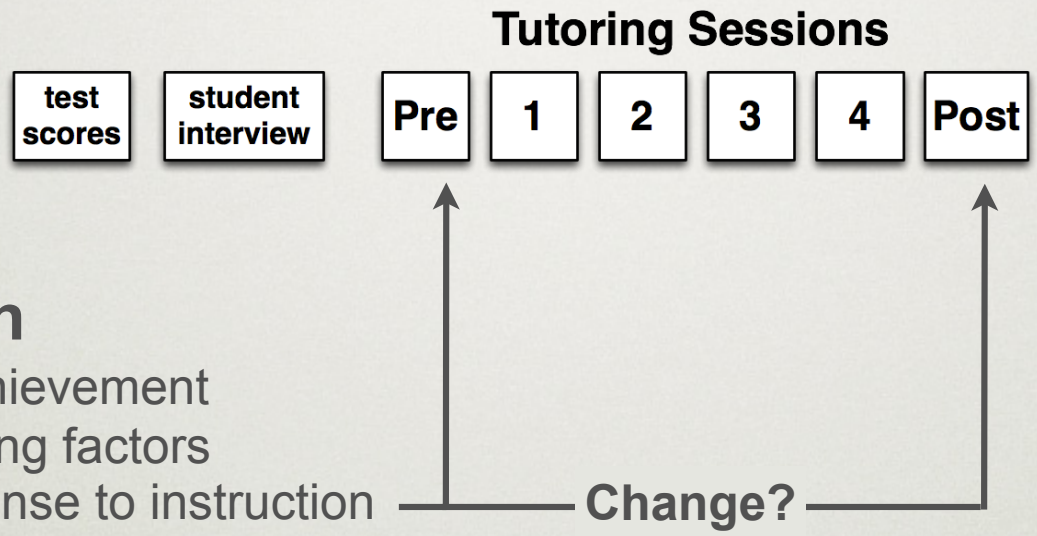
Access

Students with possible math dis.

n=11

Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction



All benefited from the tutoring sessions.

5th grade students:

- Average gain 15%
- Posttest scores above 60%

Lack of response:

- Gain less than 10%
- Posttest scores below 60%

Identification Case Study
Student with math disabilities

Students with possible math dis.
n=11

Tutoring Sessions

test scores | student interview | Pre | 1 | 2 | 3 | 4 | Post

All benefited from the tutoring sessions.

Classification

- Low math achievement
- No confounding factors
- Lack of response to instruction

2 students with a math disability
(today I will focus on one "Lisa")

Analysis of Access

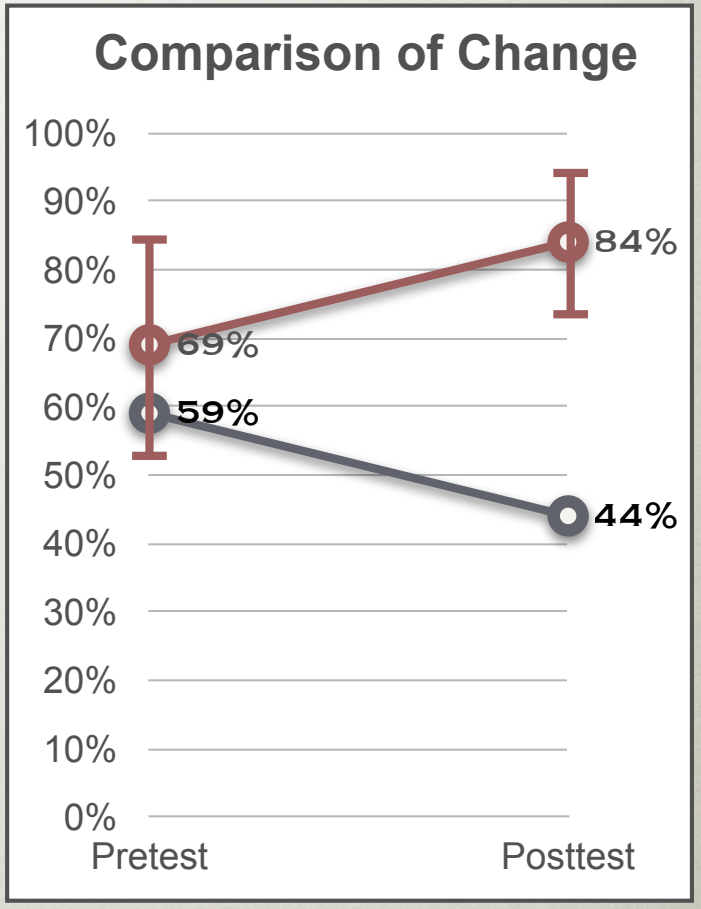
- Detailed diagnostic analysis of videotaped tutoring sessions

Lisa met Math Disability Criteria

Access

White, upper-middle class, native English speaker
Community college student.

- ☑ **Low math achievement**
  Placed in a arithmetic class - failed class
- ☑ **No confounding factors**
  No attention, behavior, anxiety or reading issues.
- ☑ **Lack of response to instruction**
  No gain from pretest to posttest.



Analyzing Access

Why did the student not benefit from the tutoring protocol?

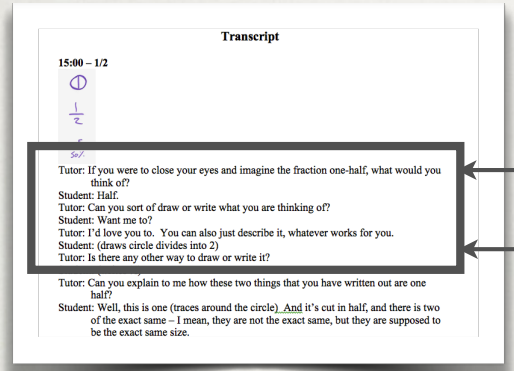
- Goals of analysis:
  - Accountable to the whole data corpus
  - Identify consequential aspects of the student's difficulty (**inaccessibility**)
  - Inaccessibility appears in an unconventional use / understanding of tools
    - **Unconventional understandings**
- Qualitative data analysis
  - Detailed microgenetic analysis (Schoenfeld, et al., 1997)
  - Analytic categories *emerge* from the data

Analytic Methods

Access

Video Data

Transcript



Transcript

15:00 – 1/2

Tutor: If you were to close your eyes and imagine the fraction one-half, what would you think of?
Student: Half.
Tutor: Can you sort of draw or write what you are thinking of?
Student: Want me to?
Tutor: I'd love you to. You can also just describe it, whatever works for you.
Student: (draws circle divides into 2)
Tutor: Is there any other way to draw or write it?

Tutor: Can you explain to me how these two things that you have written out are one half?
Student: Well, this is one (traces around the circle). And it's cut in half, and there is two of the exact same – I mean, they are not the exact same, but they are supposed to be the exact same size.

Problem Instances

Question

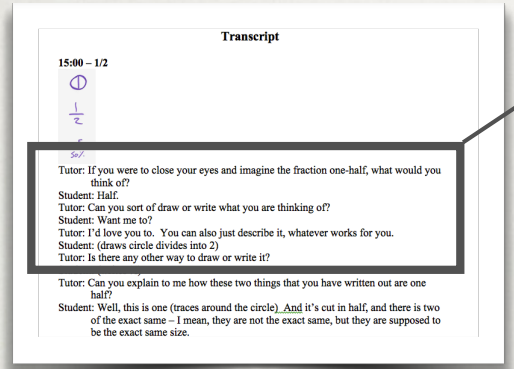
Answer

Candidate Unconventional Understandings

Analytic Methods

Video Data

Transcript



Transcript

15:00 – 1/2

Tutor: If you were to close your eyes and imagine the fraction one-half, what would you think of?
Student: Half.
Tutor: Can you sort of draw or write what you are thinking of?
Student: Want me to?
Tutor: I'd love you to. You can also just describe it, whatever works for you.
Student: (draws circle divides into 2)
Tutor: Is there any other way to draw or write it?

Tutor: Can you explain to me how these two things that you have written out are one half?
Student: Well, this is one (traces around the circle). And it's cut in half, and there is two of the exact same – I mean, they are not the exact same, but they are supposed to be the exact same size.

Candidate Unconventional Understandings

Problem Instances

Correctness 

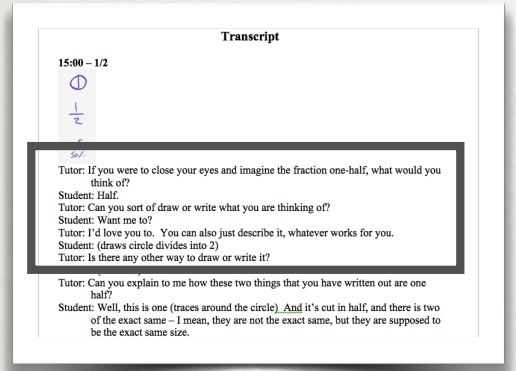
Evidence of unconventional understandings

Analytic Methods

Access

Video Data

Transcript



Transcript

15:00 – 1/2

Tutor: If you were to close your eyes and imagine the fraction one-half, what would you think of?
Student: Half.
Tutor: Can you sort of draw or write what you are thinking of?
Student: Want me to?
Tutor: I'd love you to. You can also just describe it, whatever works for you.
Student: (draws circle divides into 2)
Tutor: Is there any other way to draw or write it?

Tutor: Can you explain to me how these two things that you have written out are one half?
Student: Well, this is one (traces around the circle). And it's cut in half, and there is two of the exact same – I mean, they are not the exact same, but they are supposed to be the exact same size.

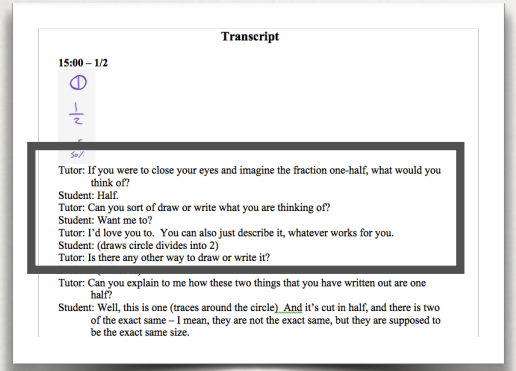
Candidate Unconventional Understandings

Problem Instances

Analytic Methods

Video Data

Transcript



Transcript

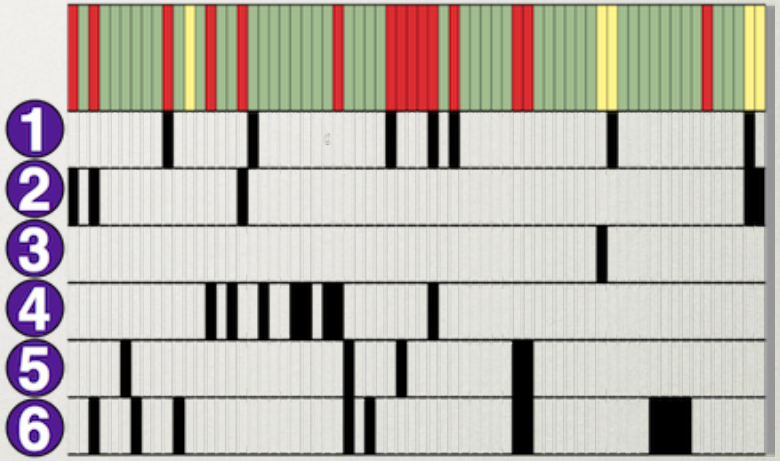
15:00 – 1/2

Tutor: If you were to close your eyes and imagine the fraction one-half, what would you think of?
Student: Half.
Tutor: Can you sort of draw or write what you are thinking of?
Student: Want me to?
Tutor: I'd love you to. You can also just describe it, whatever works for you.
Student: (draws circle divides into 2)
Tutor: Is there any other way to draw or write it?

Tutor: Can you explain to me how these two things that you have written out are one half?
Student: Well, this is one (traces around the circle). And it's cut in half, and there is two of the exact same – I mean, they are not the exact same, but they are supposed to be the exact same size.

Candidate Unconventional Understandings

Problem Instances

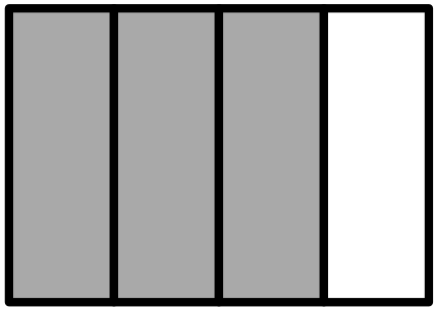


All transcript accounted for in coding system

Lisa's Taking Understanding

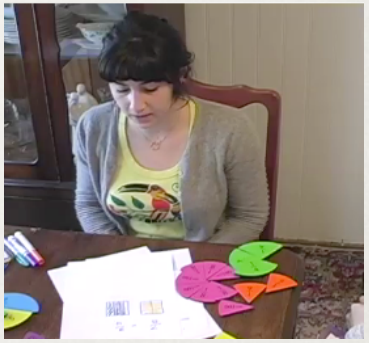
draw $\frac{3}{4}$

"taken" "left"

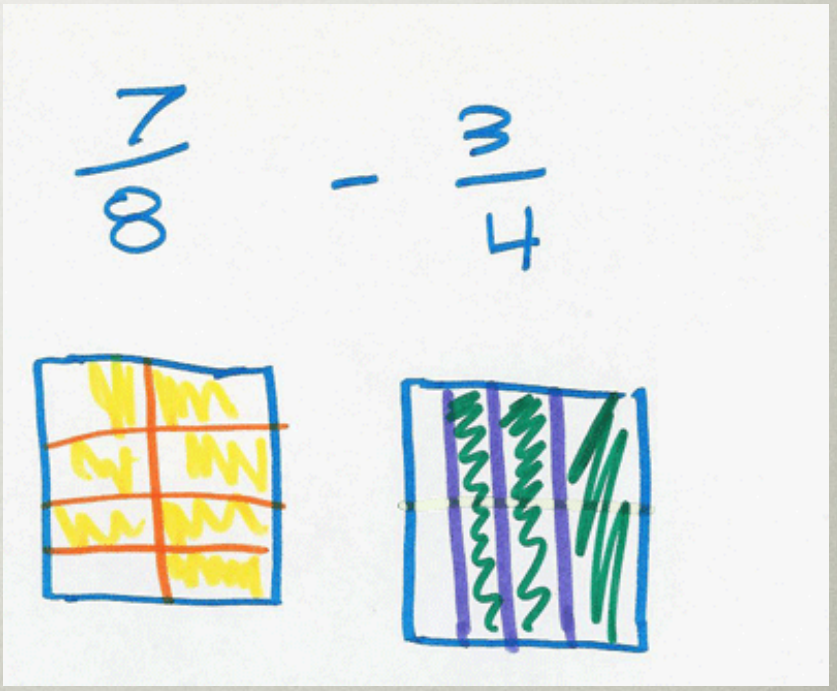


$= \frac{3}{4}$

Unconventional focus on the fractional complement



$\frac{7}{8} - \frac{3}{4} =$



Transcript
Question: 7/8-3/4 =

Katie: Ok, so now, how do we think about the relationship of this to this (point to each area model)? Because we want to basically take this (pointing to 3/4 area model) amount away from here (pointing to 7/8 area model).

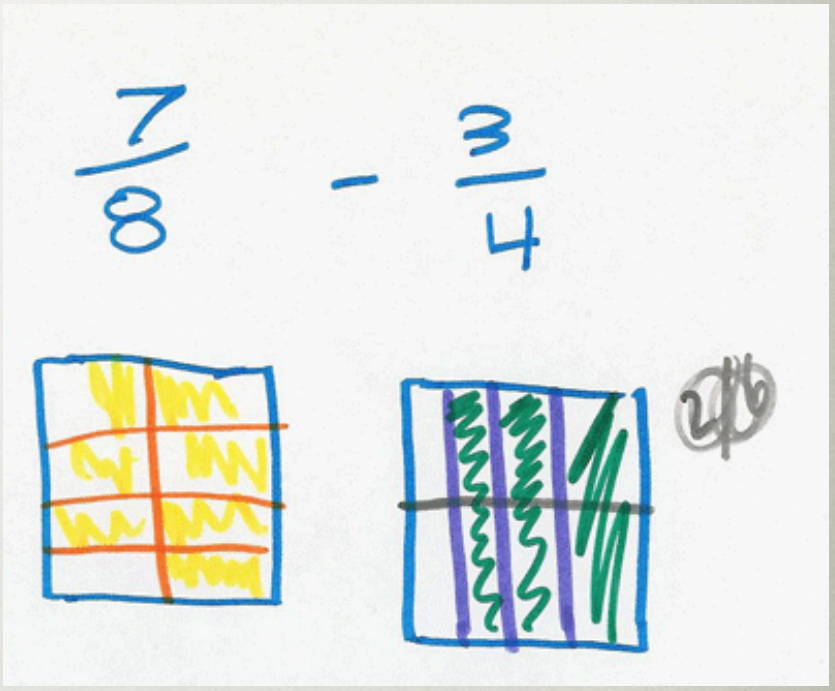
Lisa: If I were to switch this (gestures horizontal partition line over 3/4) Like that

Katie: Ok.

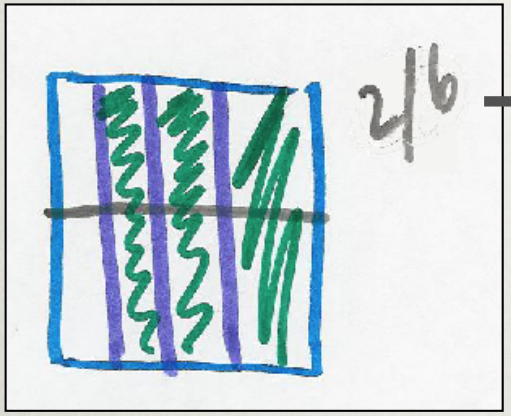
Lisa: it would be 2….

Katie: So let's cut it in half like that.

Lisa: Can I use this? (divides shape) there would be two left? Or, two out of... Two-sixths left (writes 2/6)? No. I'm not sure. (scribbles out 2/6)



Fractional complement



$\rightarrow \frac{2}{8}$

"two left"
shaded as "taken"

"A third...?" "How much is left?"

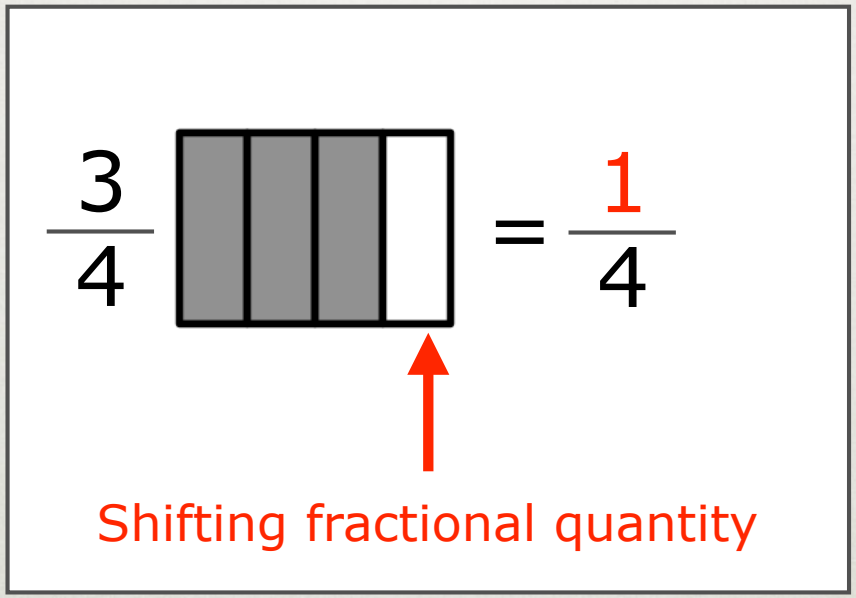
"Yeah, a third is missing."

- Lisa attended to the fractional complement across various representations (mediational tools)

Unconventional Understanding issues of Access

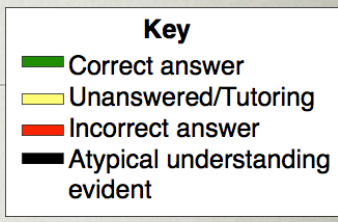
Access

Fractional Complement

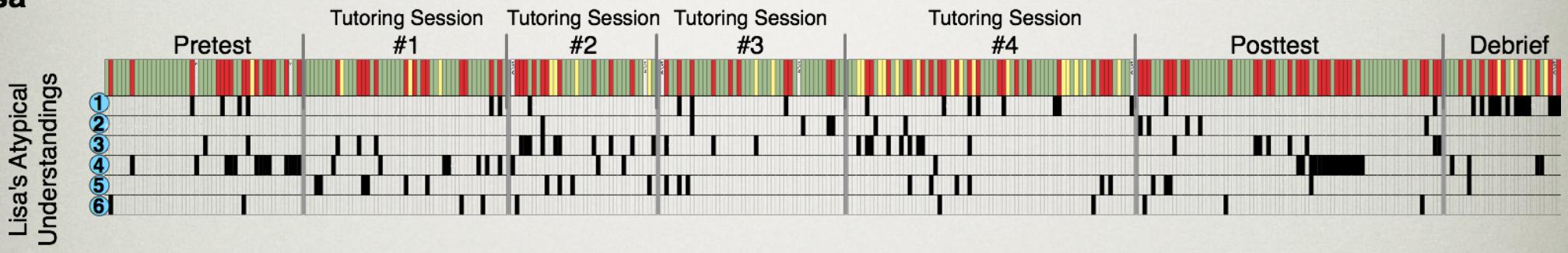


- Representations of fractions (like area models) do not serve same purpose for Lisa. (unconventional understanding)
- Rather than representing quantity - they seem to represent action. (issue of access)

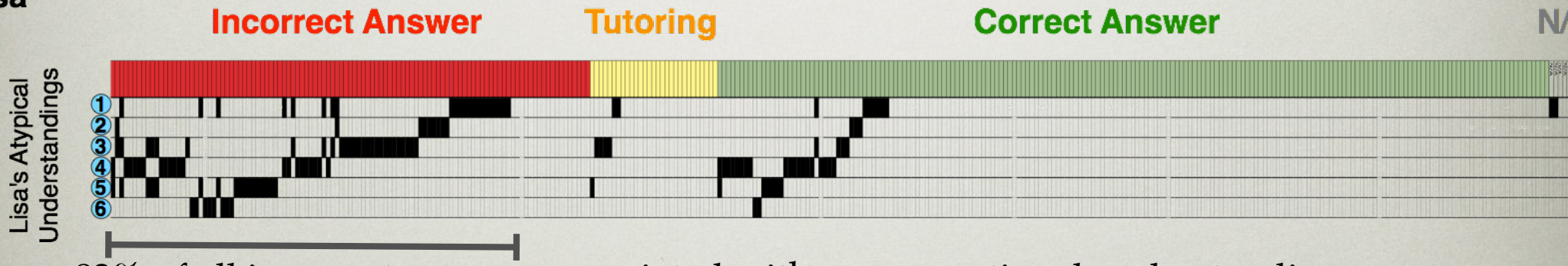
OVERVIEW OF DATA



Lisa



Lisa



82% of all incorrect answers associated with unconventional understandings

Persistent **Robust** **Detrimental**

Lisa's Understanding

Tutoring Sessions

Pre	1	2	3	4	Post

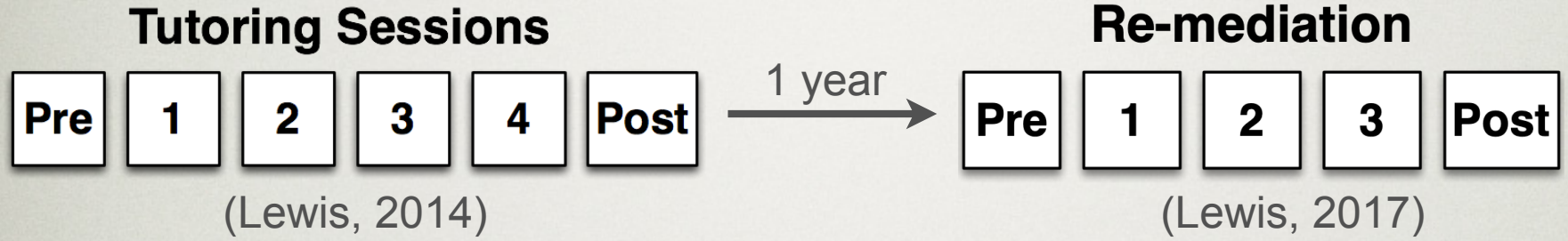
(Lewis, 2014)

- **Standard representations** were at least partially inaccessible.
- Lisa's unconventional "Taking" understanding was not compatible with canonical usage of this mediational tool.

(Lewis, 2017)

Re-mediation

Re-mediation



(Lewis, 2014)

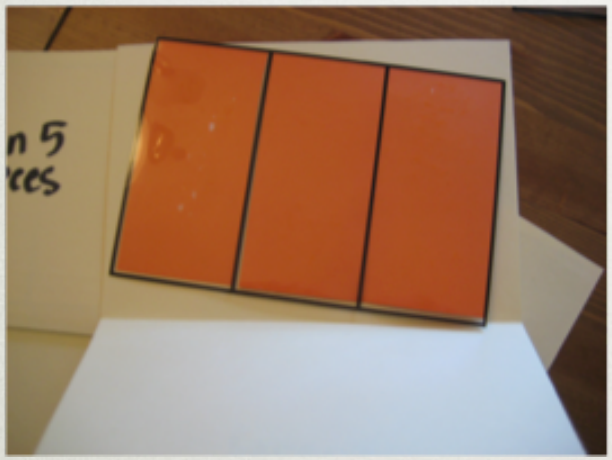
(Lewis, 2017)

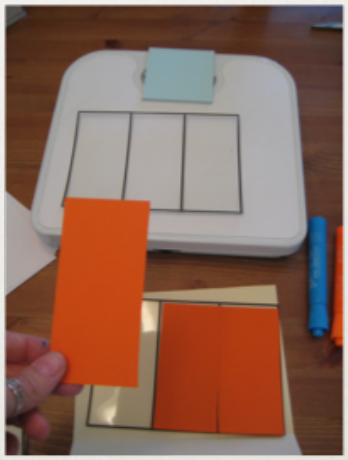
- **Standard representations** were at least partially inaccessible.
- Lisa's unconventional "Taking" understanding was not compatible with canonical usage of this mediational tool.

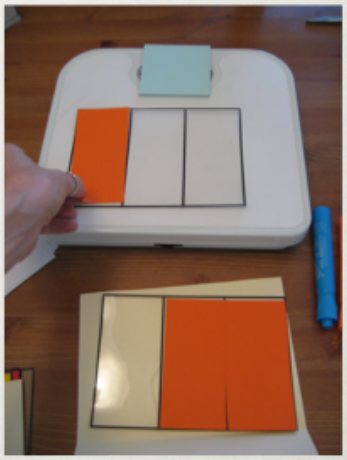
- Build upon her "taking" understanding
- Design a re-mediation which enables her "taking" understanding to be a productive
- Connect to the canonical mathematical usage of area models

Scale Re-mediation

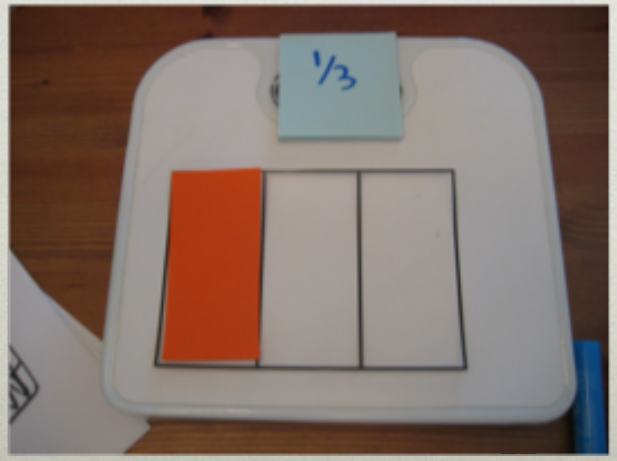
Allow shaded to be “taken”

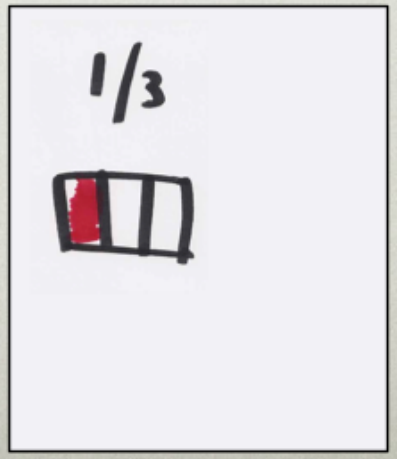




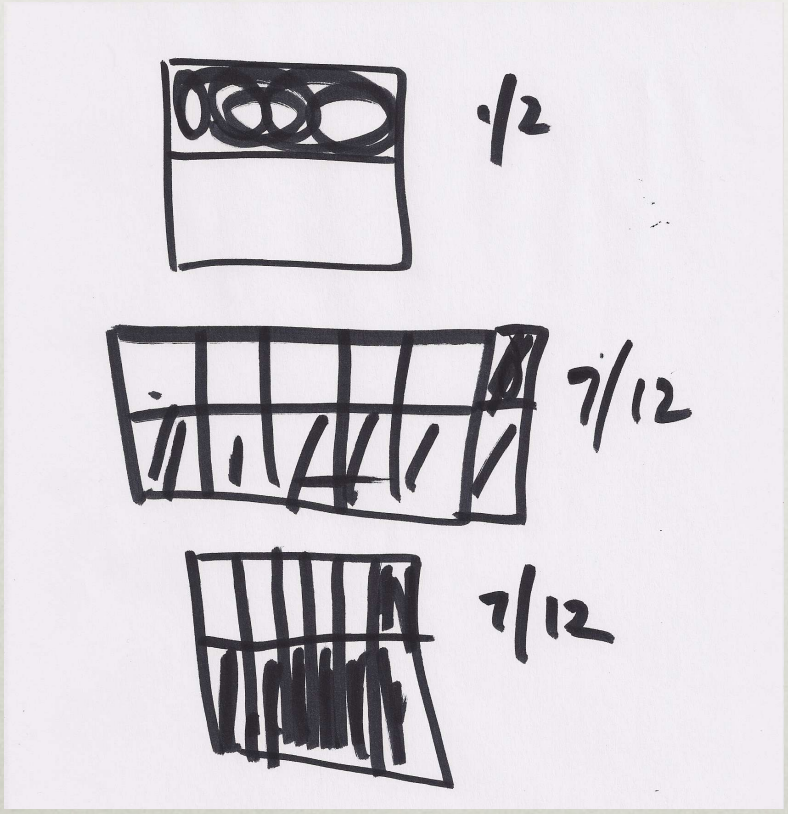


Refocus on fractional quantity





A year later Does Lisa still rely on her Taking Understanding?



- Listen for:
  - Lisa referring to the shaded region as an amount that has "gone away"
  - The non-shaded pieces as the amount "left"

Transcript pretest
Compare 1/2 and 7/12

Katie: Do these pictures help you think about that (referring to assertion that ½ is larger than 7/12)?

Lisa: Yeah.

Katie: And how, by looking at these pictures. Can you explain that one-half is bigger than seven-twelfths? Like are things. Are there things that you can point to about it?

Lisa: Here, let me, let me do it this way. (draws another picture of 7/12 below) Ok, so this is one half. It's like the same bread or whatever.

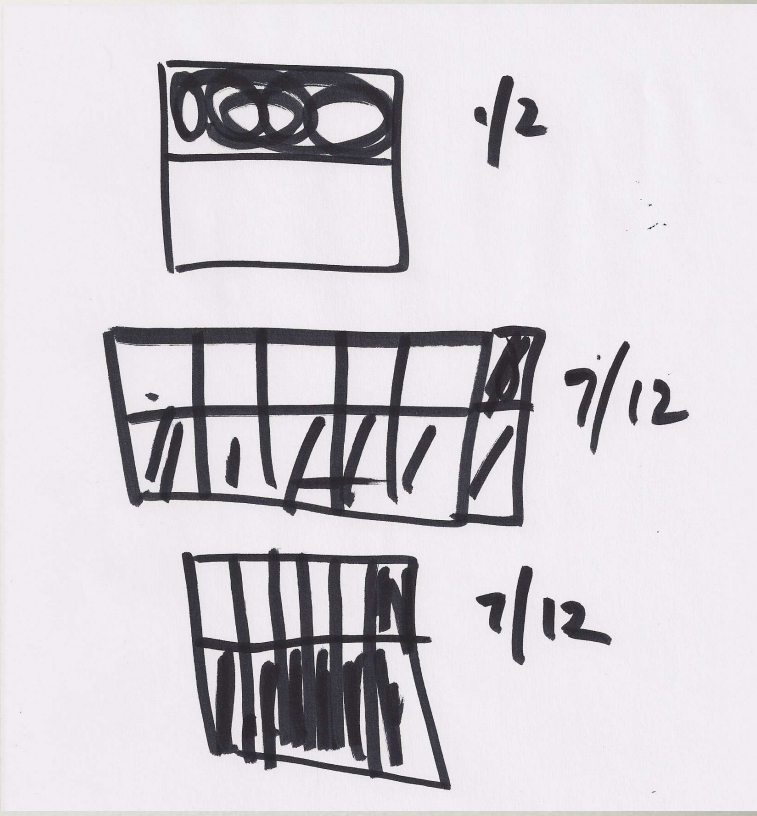
Katie: Mm hmm.

Lisa: Um, someone would want to opt for a half of the bread or whatever than 7/12 the bread, because at the end of the day, all of this would go away, leaving one, two, three, four or five (pointing at the unshaded pieces) pieces left. That are small, because it's been sectioned off 12 times. whereas the other person would want half of the entire loaf.

Katie: Okay. Okay. So in the comparison, you're sort of comparing this big piece to those five pieces there?

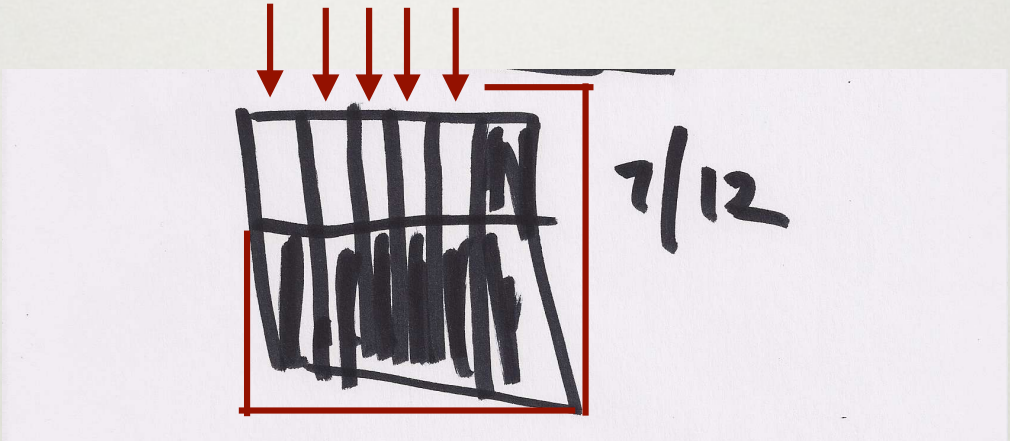
Lisa: Yes.

Katie: Okay.



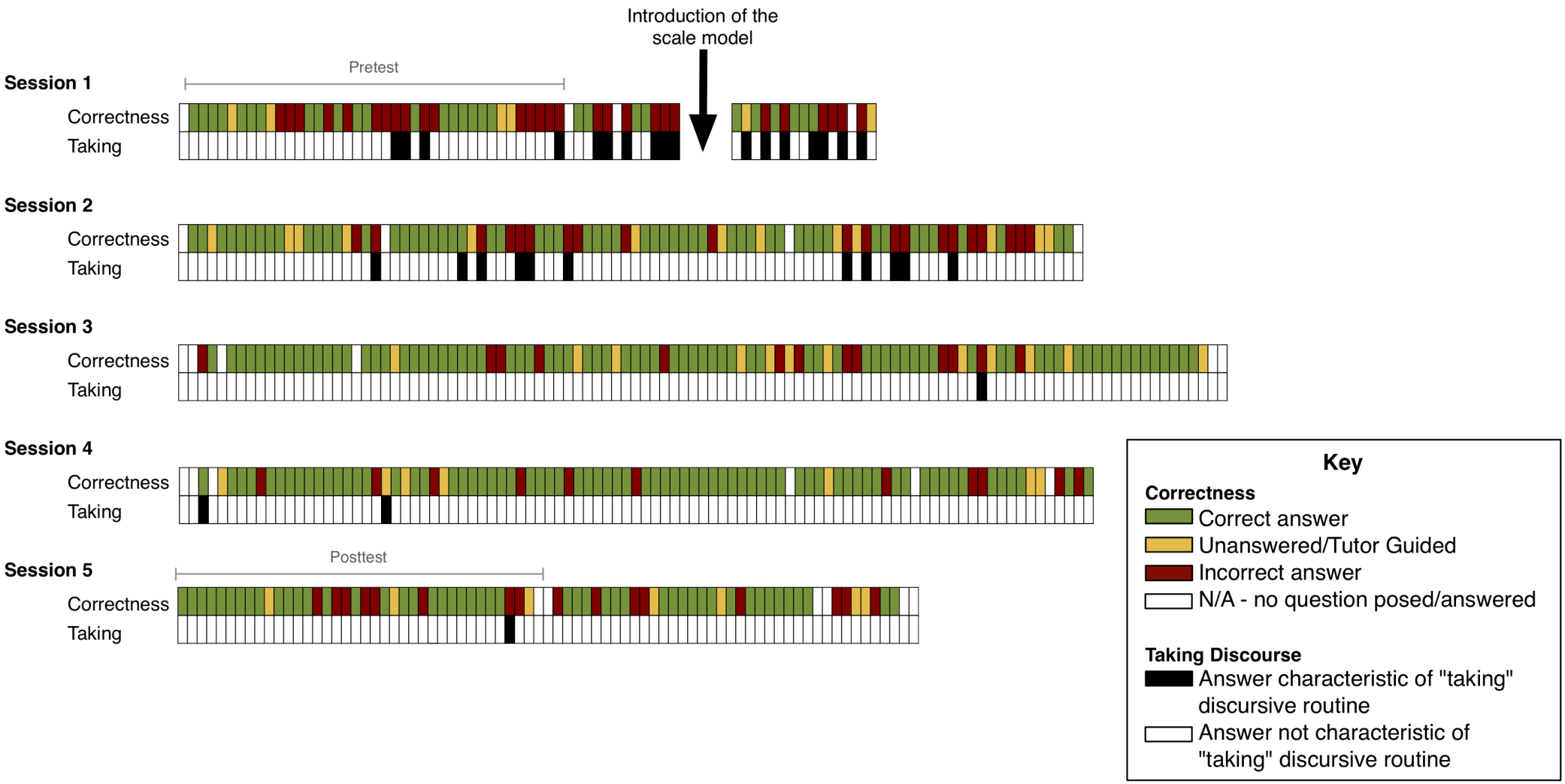
Lisa's taking understanding

"leaving 1, 2, 3, 4, 5 pieces left"



"all of these would go away"

Overview of re-mediation



Effectiveness of Remediation

Re-mediation

Remediation

Pre | 1 | 2 | 3 | Post

Before Remediation

"pieces taken"

1

"pieces left"

3

After Remediation

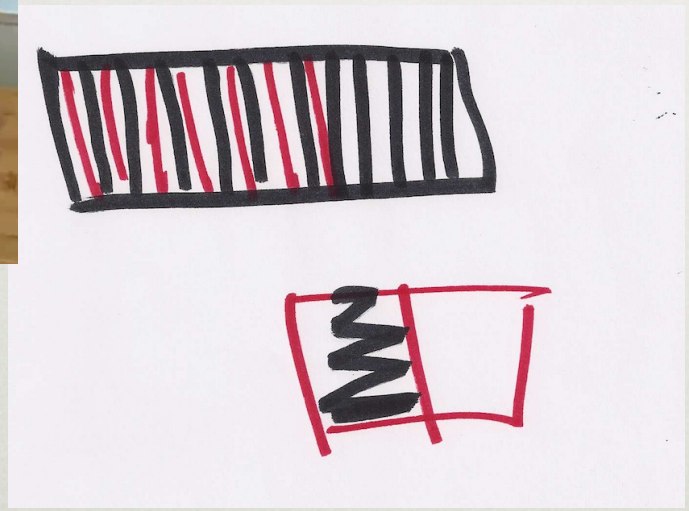
"pieces there"

"pieces weighed"

$\frac{2}{3}$

"pieces not there"

Posttest
Compare 1/2 and 7/12



- In this clip
  - Lisa had decided that 1/2 was larger than 7/12 and decides to draw them.

Listen for:

- Her focus on the size of the pieces.
- When I ask her to think in terms of the scale how she shifts her thinking

Posttest Transcript
Compare 1/2 and 7/12

Lisa: Yeah. I'm still gonna say one half is probably larger. Okay. Um, these are smaller fractions of a whole. So each piece is smaller. Et cetera.

… (student attempts and is not sure how to order 9/10) …

Katie: And does thinking of the scale at all help you sort of decide which of these two (pointing to 7/12 and ½) is bigger?

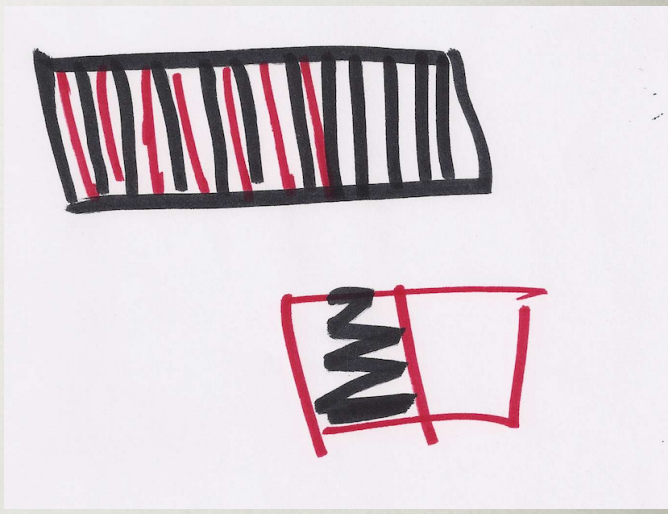
Lisa: Kind of. Yeah. Yeah.

Katie: Okay. So if these were pictures of the scale?

Lisa: I guess if we were looking at the scale, this (pointing at 7/12 area model) would be heavier.

Katie: Okay.

Lisa: 7/12 would be heavier. Yeah. It's hard to say for sure. Like, I'm not certain about any of this, but I guess if I do, like sit there and visualize it, like the seven pieces, although they're... They're thinner, I feel like there would be more of them that's making it heavier.

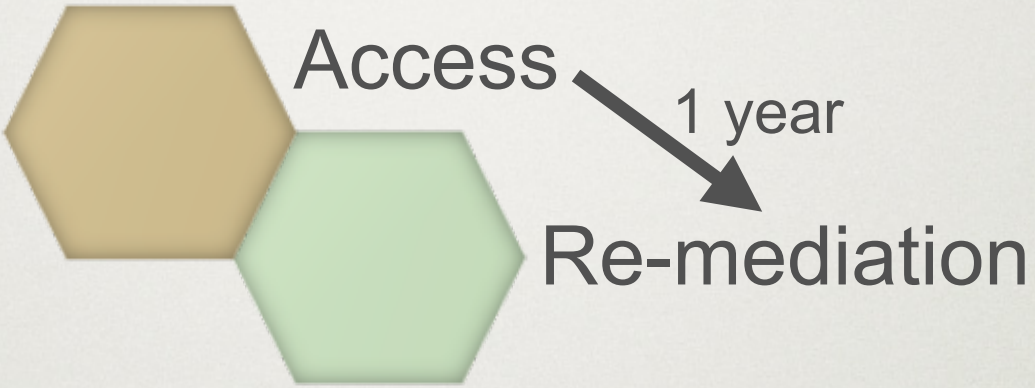


What is the mathematical shift?

Re-mediation

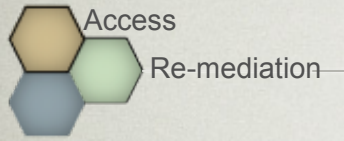
- She did initially believe that 1/2 was larger than 7/12
- **BUT**…
  - She was not comparing the fractions based on the amount "left"
  - When asked to think about the scale she revised her answer
  - She coordinated both the meaning of the numerator and denominator

Lisa's Case

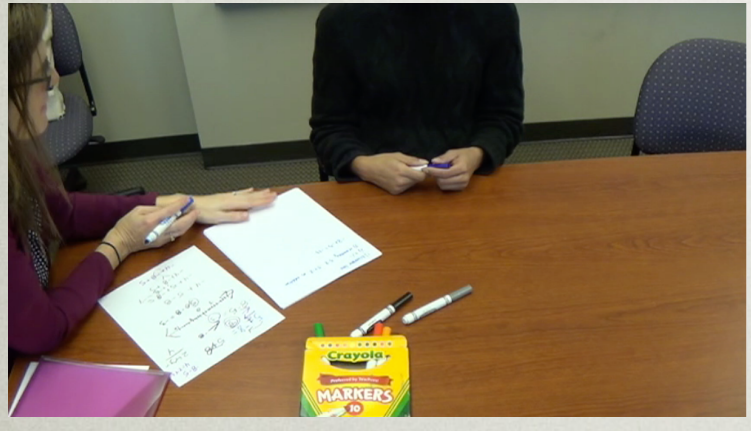


- Lisa had **unconventional understanding** of standard meditational tools
- Standard meditational tools were at least partially **inaccessible** for Lisa
- Lisa was able to achieve greater access through alternative meditational tools (**re-mediation**)

Case #2 - Melissa

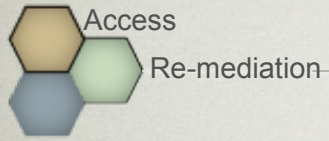


(Lewis et al., 2020; 2022)

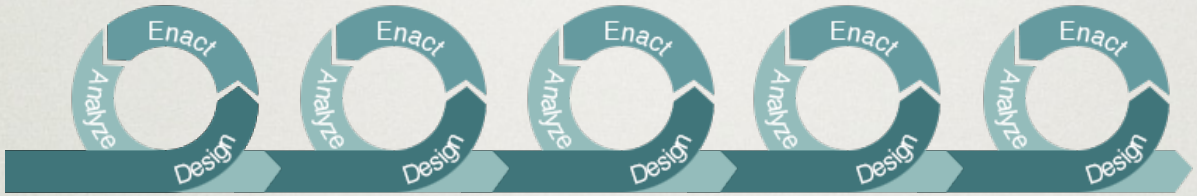


- "Melissa" - 31 year old, half Black half White, community college student, history of math difficulties
- (19th percentile on Woodcock Johnson)

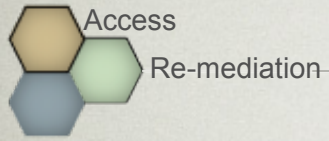
Design Based Research Approach



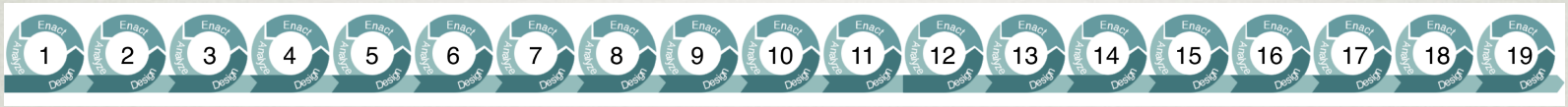
- Design Microcycles (Gravemeijer & Cobb, 2006)



Design Based Research Approach



- Design Microcycles (Gravemeijer & Cobb, 2006)

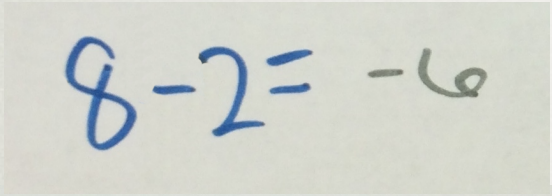


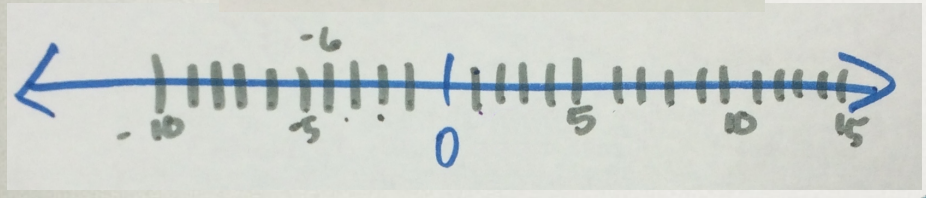
Retrospective Analysis

- 19 sessions focused on integer operations (Lewis et al, 2020) and solving for unknowns (Lewis et al, 2022)

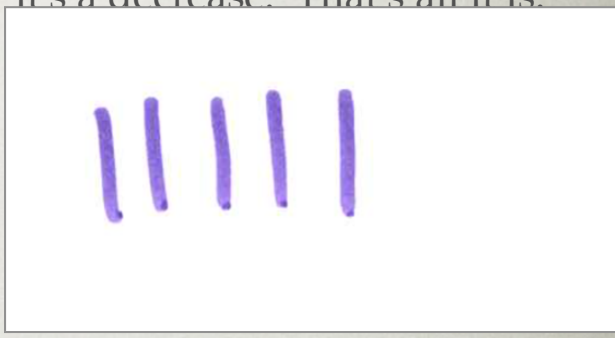
Integers

Access
Re-mediation





Describe -4
Describe 5



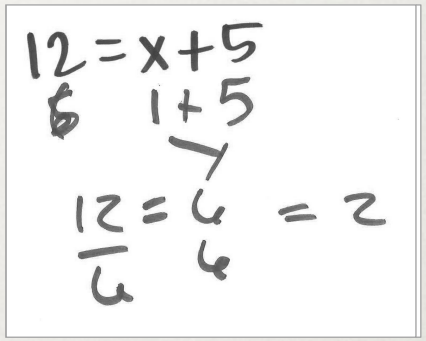
-4 is not an object
5 is not an object
Quantities are understood as processes not objects

Represent quantities with manipulatives

Algebra (Solving for x)

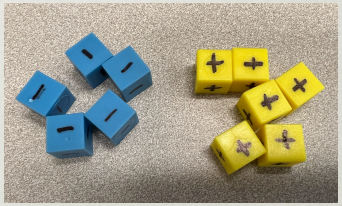
Access
Re-mediation

12=x+5 simplified to 12=1+5

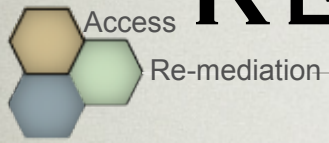


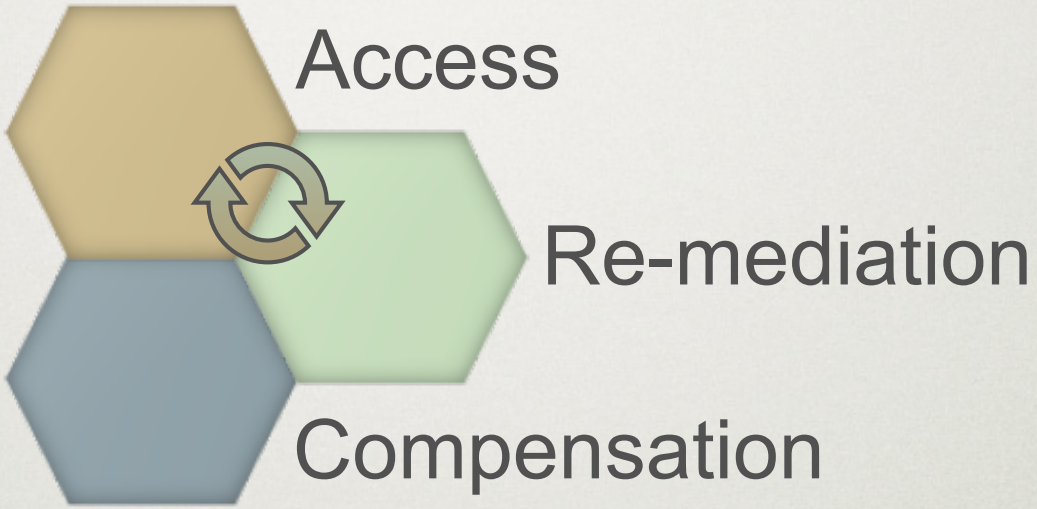
Thinks of x as 1
Symbolic notation (x) may be inaccessible

Ask student to physically represent unknowns as container with a value contained within



Features of Design Based Research Re-mediations





- Melissa had issues of access related to her understanding of **quantity** and **notation** for both integers and algebra.
  - Both re-mediations involved a physical representation of quantities
  - Both required translation of inaccessible notation
- Design-based research re-medations enabled us to identify issues of access and design alternative meditational tools.
- Reorientation to the tools was effective.

Case #3 - Dylan Lynn

Compensation

Compensation

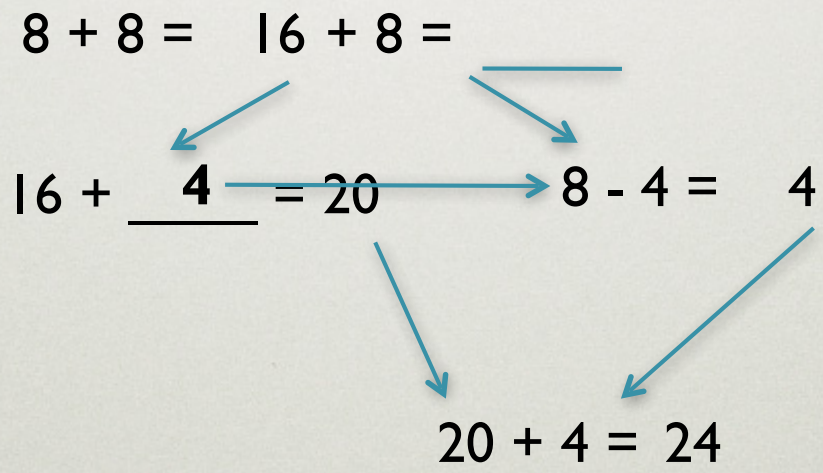
(Lewis & Lynn, 2018)

Solve the problem:

8x3 =

Compensation

- 10.11 seconds to solve
- Process



8 x 3 = 24

Dylan's Story

"if I have a math learning disability then the accommodation is to not take math."

Revisiting theoretical Framing

- Disability Studies (moving from social model to political model)
  - Traditional research *on* individuals with disabilities is oppressive.
  - "Researchers have benefitted by taking the experience of disability, rendering a faithful account of it and then moving on to better things while the disabled subjects remain in exactly the same social situation they did before the research began" (Oliver, 1992, p. 109).

Revisiting theoretical Framing

- Inverts the power dynamic (Mercer, 2004; Oliver, 1992)
- Researchers place their knowledge at the disposal of individuals with disabilities (Barton, 2006).
- Dylan = expert and Katie = inquirer (see Knox, Mok, & Parmenter, 2000)
- Individuals with disability decide
  - Agenda
  - Methodology
  - Interpretation
  - Final product

Dylan's Agenda

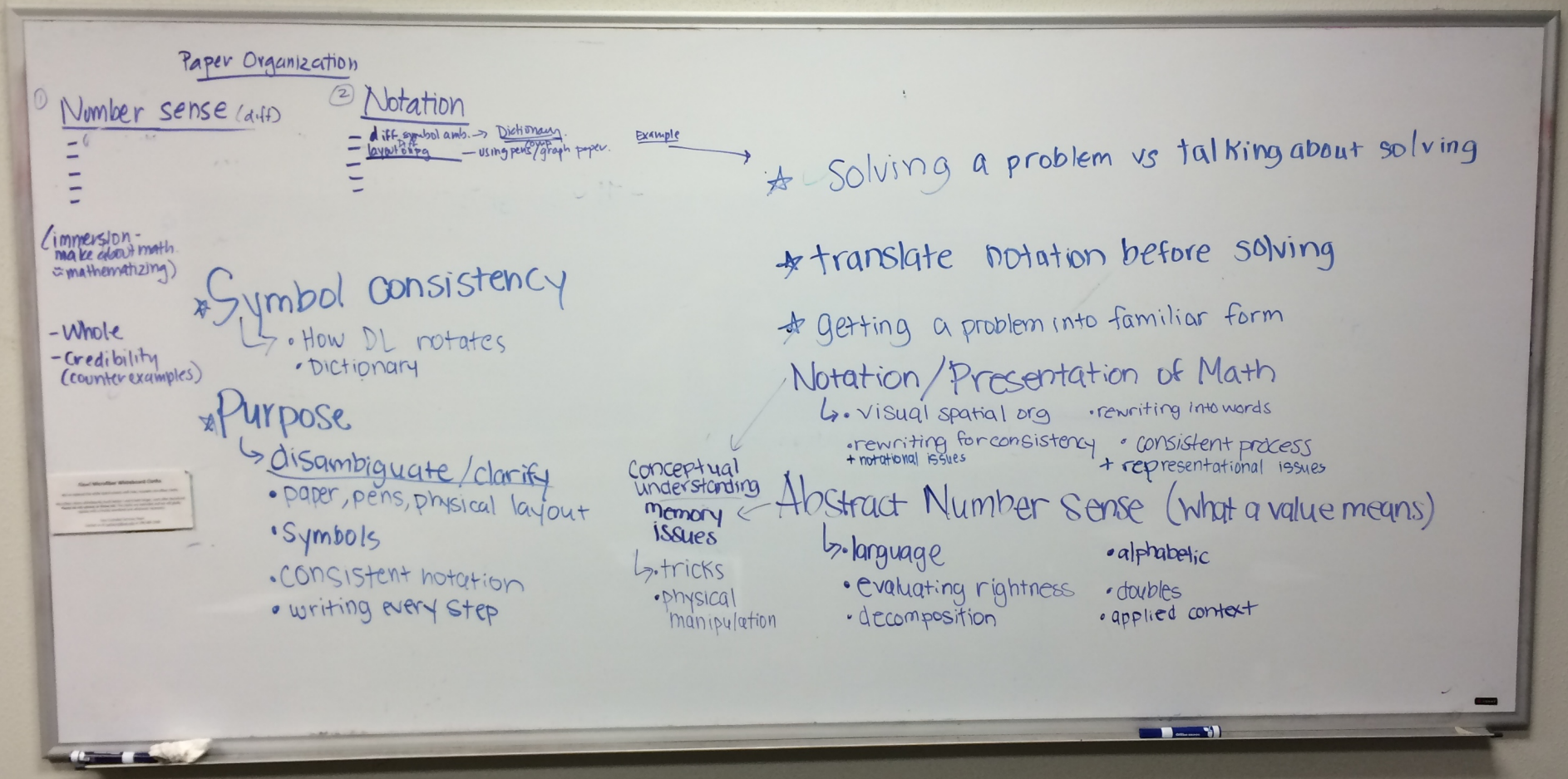
- Document and share Dylan's strategies
- "I'm doing it in the hopes that having this kind of knowledge can help younger people, especially young kids in particular, with being able to pursue higher-level mathematics, and not facing the difficulties I did."

Methodology

- Videotaped conversations with Katie and Dylan
  - Work through agenda focusing on Dylan's difficulties and ways of compensating for various mathematical topics
  - At the end of each session we planned agenda for next session.
  - Katie wrote up notes, Dylan reviewed and

Interpretation – Analysis

- Reviewed video and transcripts of sessions to identify categories.
- Refined these categories drawing extensively on Dylan's expertise and interpretation.


Paper Organization
① Number sense (diff)
② Notation
diff. symbol amb. → Dictionary
layout org — using pens/graph paper
Example
Solving a problem vs talking about solving
translate notation before solving
getting a problem into familiar form
(immersion - make about math = mathematizing)
- Whole
- Credibility (counter examples)
Symbol consistency
• How DL rotates
• Dictionary
Purpose
disambiguate/clarify
• paper, pens, physical layout
• Symbols
• consistent notation
• writing every step
Notation/Presentation of Math
• visual spatial org
• rewriting for consistency + notational issues
• rewriting into words
• consistent process + representational issues
Conceptual understanding
memory issues
Abstract Number Sense (what a value means)
language
• evaluating rightness
• decomposition
• alphabetic
• doubles
• applied context
tricks
• physical manipulation

Compensation

- Our work tries to capture Dylan's compensatory strategies
- Compensation = deliberate and intentional (re)action to inaccessible context in order to gain access.
- (difficulties—> access issues —> compensation)

- Two kinds of rewriting
  - Rewriting to in words
  - Rewriting to remove ambiguity

#1 - Rewriting in words

- Difficulty remembering numerical information and "reading" mathematical notation
- "Like sounding out the math…"

$$3+2=$$

Three plus two is ___?

$$P(A|B)$$

Probability of event A happening, given event B has happened

#1 - Rewriting in words

3+2=

Dylan: I started thinking about the word rather than the symbol. Because the word like T H R E E has much more meaning to me than that too little backwards C's laying on top of each other.

#1 - Rewriting in words

P(A|B)

Dylan: Something like this is much shorter than me for me now, but in my higher division math courses, I will actually still write these things out because you know,

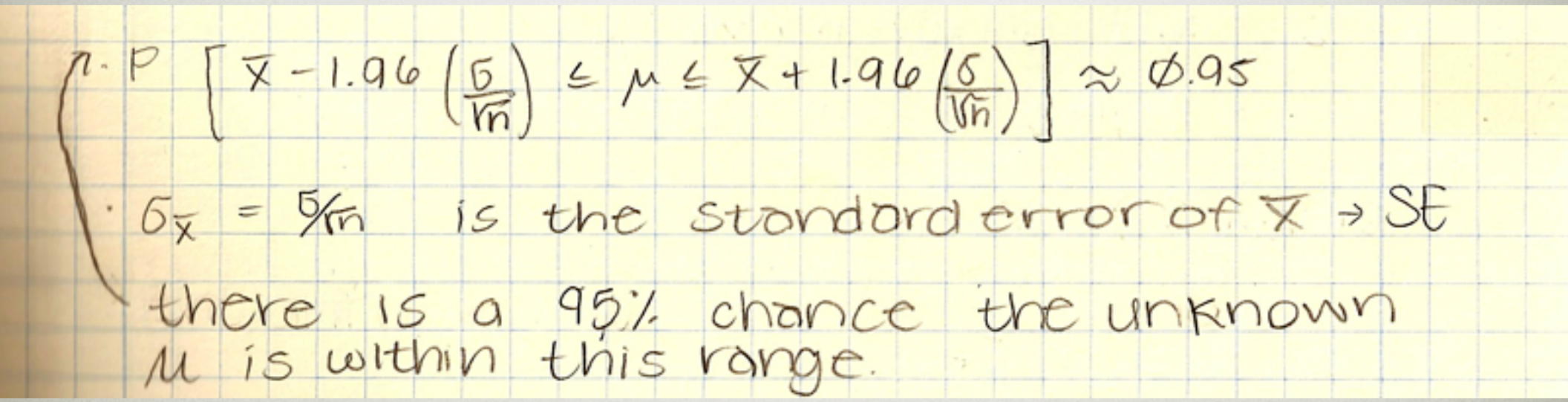
Katie: like the probability?

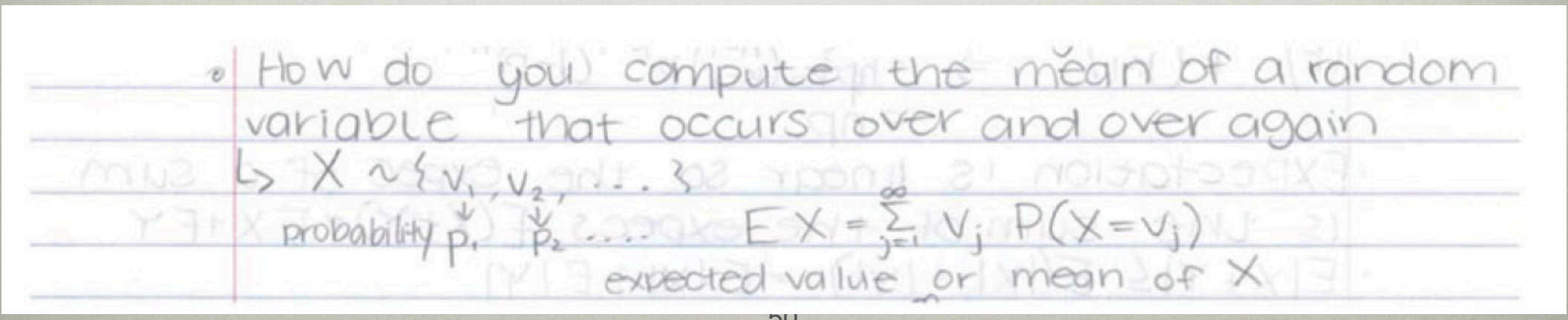
Dylan: Yeah, this is so incredibly short, but it actually translates to the probability of event ha--event A happening given event B has happened. I actually have to like physically write that every time I do one of these.

#2 - Rewriting in words

Compensation

- From notes:





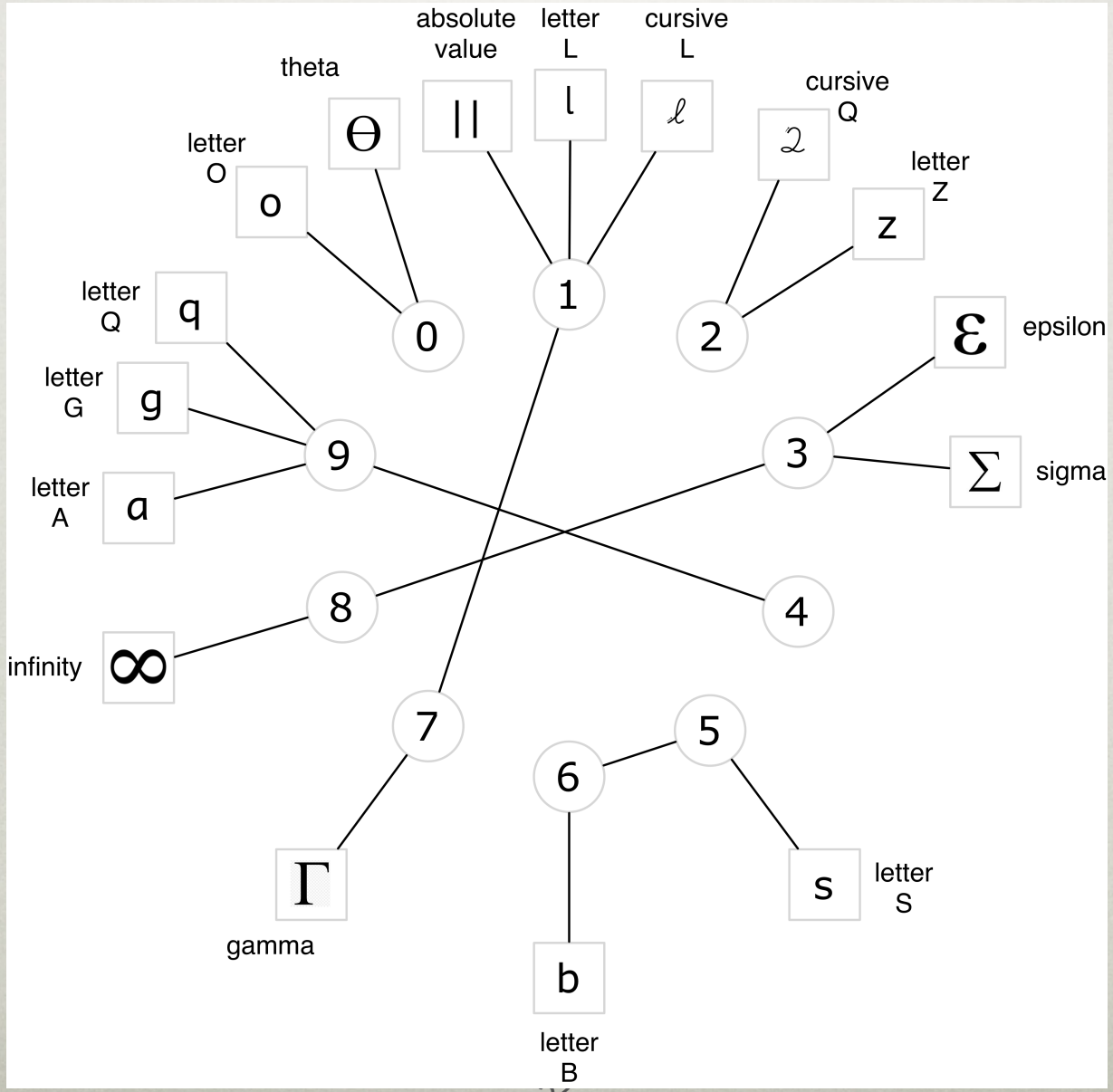
#2 - Rewriting to remove ambiguity

Compensation

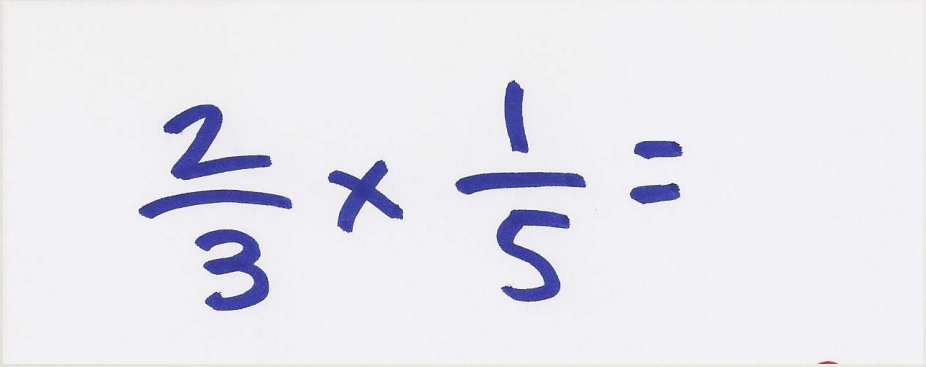
- Difficulty with symbols that look the same

- Difficulty with symbols that can mean different things
  - 3**x**5= and 3**x**+9=27

SIMILARITY CREATED ACCESSIBILITY ISSUES

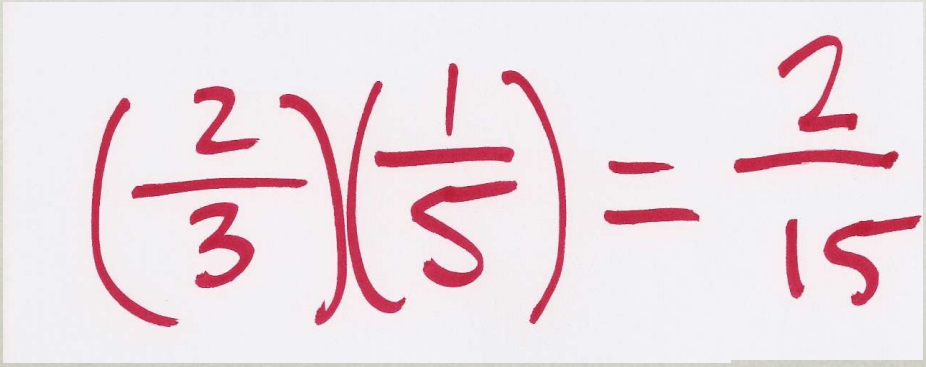


#2 Rewriting to remove ambiguity



Representational ambiguity

- "Because I have taken algebra and I know that *x* can in fact be a variable and not necessarily multiplication… I always use parentheses now for multiplication."



Rewritten Problem

Benefits of rewriting

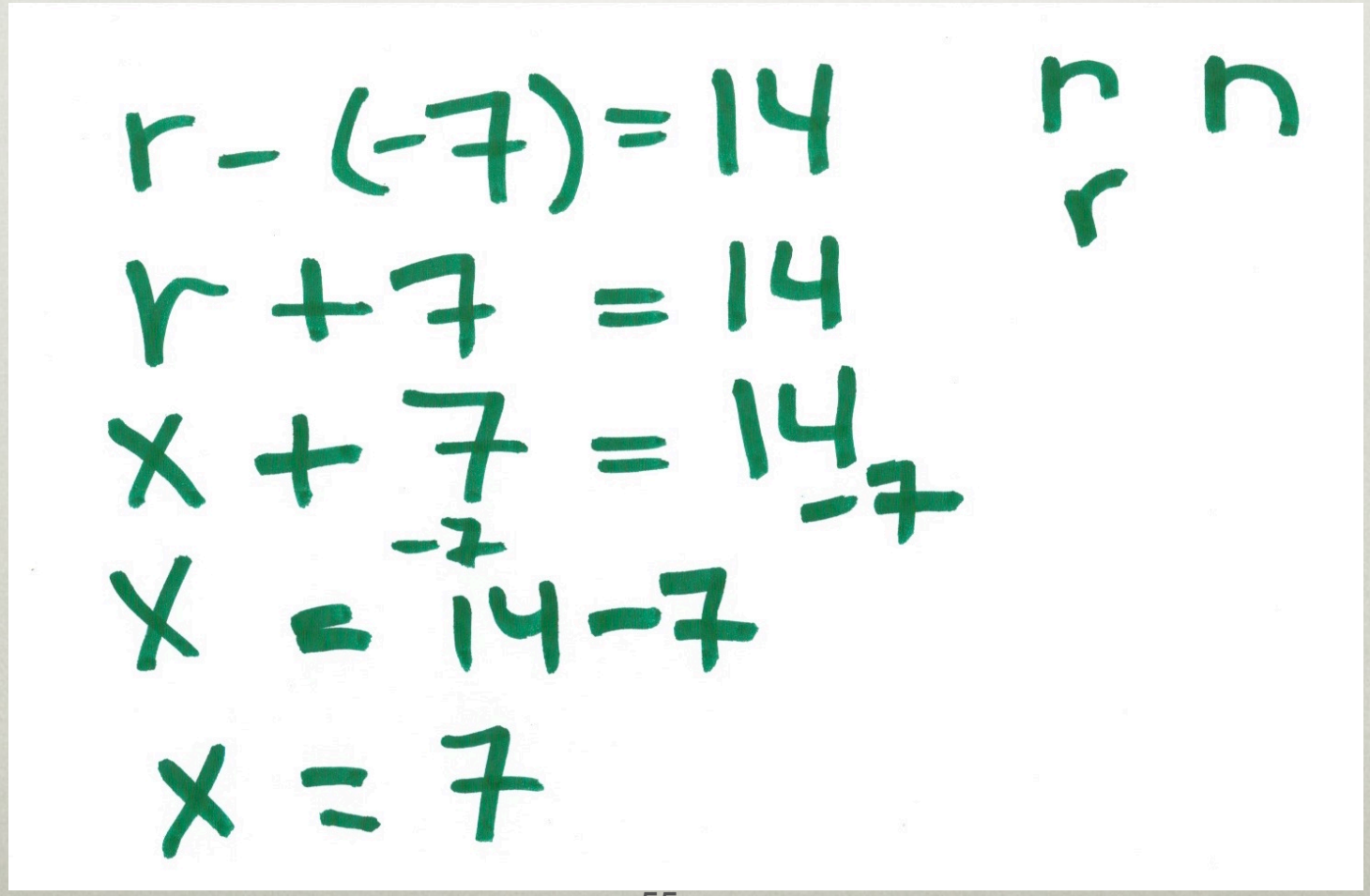
- Enables association of the meaning with the notation
- Enables translation into a known form

- **Access to the mathematics!**

Demands of Rewriting

Compensation

- Time consuming
-



Demands of Rewriting

- Time consuming
- Need to integrate new symbols into "dictionary"

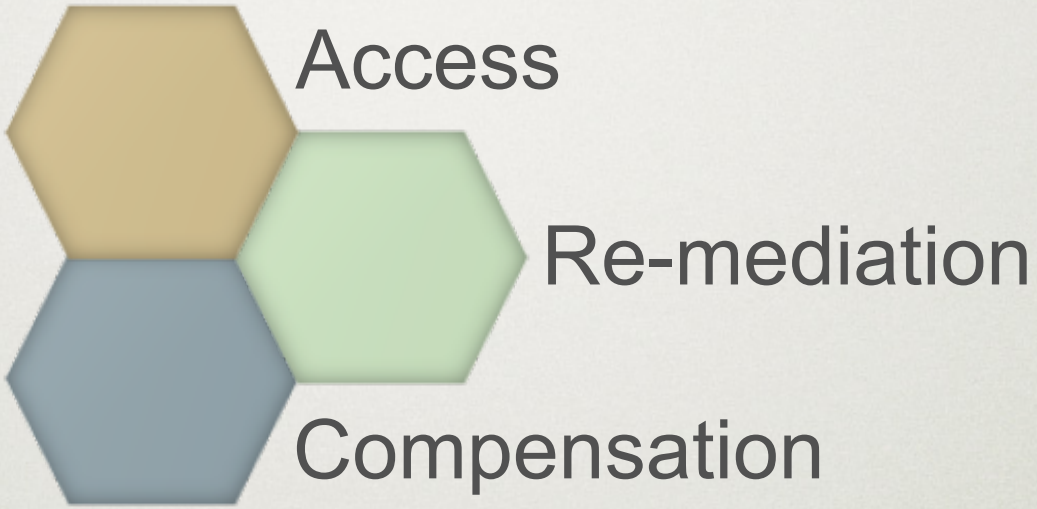
Compensation

- It is possible to compensate even if the individual experiences fundamental issues with processing symbols.
- Strategies that Dylan shared we have used in our DBR work with other students (e.g., Melissa's notational issues were addressed by rewriting in words 8-2= or 12=x+5)

Emancipatory Work

- Raises questions about power dynamics in research
- Who is deciding the agenda, methodology, interpretation, final product?
- Who gets to author the participants' stories?
- Who benefits?

Anti-Deficit of Math disabilities



- Expands the ways in which we can approach the study of MLDs
- Not just quantifying deficits, but attempting to capture differences
- Using the understanding of difference (and issues of access) to design alternative re-mediations (to provide access)
- Demonstrating how it is possible to compensate, even if the compensations require more time and effort
- Emancipatory work - how we can our work be in service to our participants?

Thank you

- Participants - Lisa, Melissa, and Dylan Lynn (co-researcher)
- Co-authors - Dylan Lynn, Gwen Sweeney, Grace Thompson, Rebecca Adler, Kawla Alhamad
- Funding for this work
  - The Spencer Foundation Dissertation Fellowship
  - National Academy of Education and Spencer Postdoctoral Award
  - University of Washington - Royalty Research Fund Award
  - National Science Foundation under grant No. ESI-0119732, and
  - The Institute of Education Sciences under grant R305B090026