What Counts in Mathematics (Especially Algebra) Classrooms?

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Outline

1. Framing the big question
2. Trying to figure out "what counts"
3. The history behind our scheme
4. Using the scheme to code a video
5. Discussion

(6, if there's time: Ask me about PD)

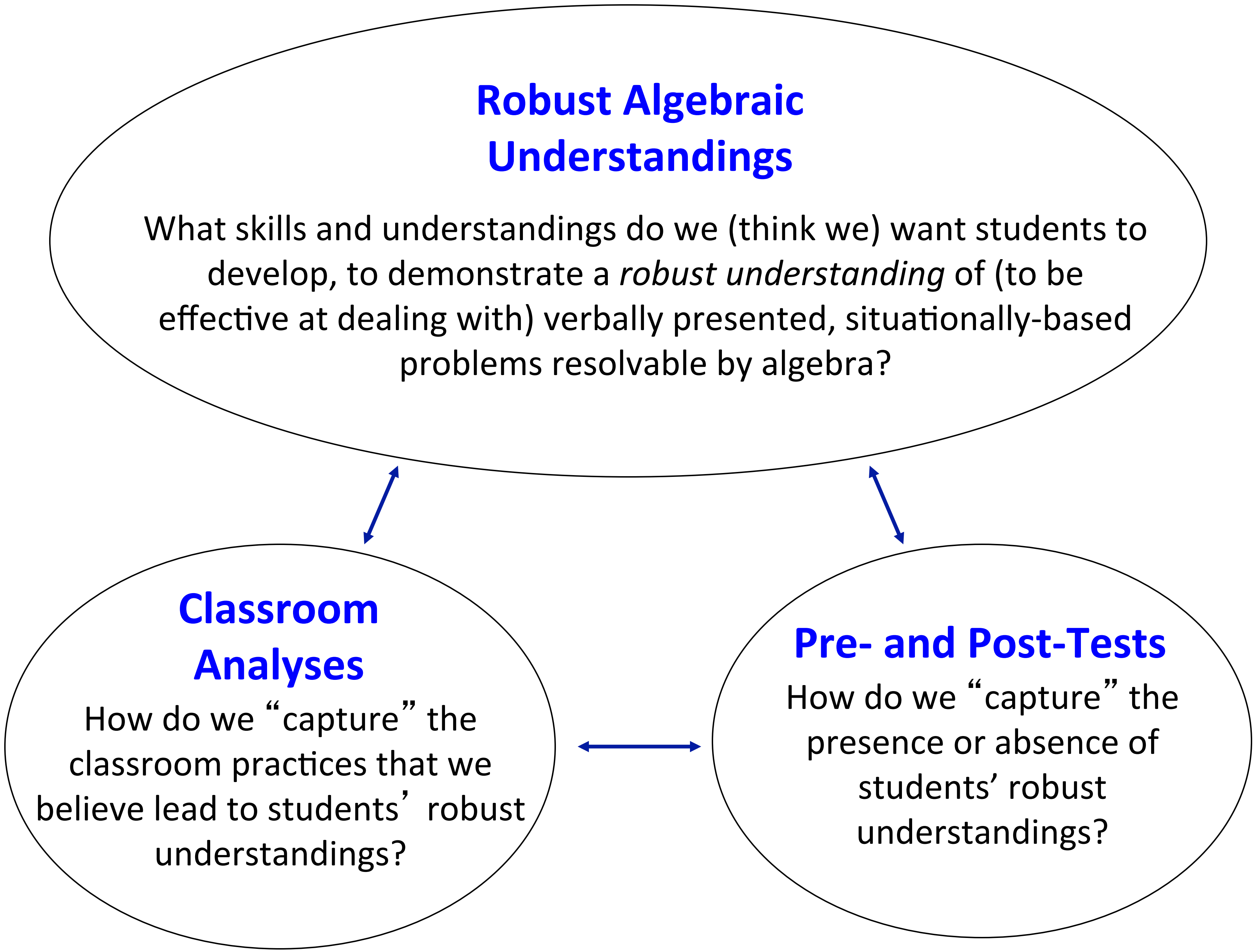
Part 1

Framing the big question

The Algebra Teaching Study

Alan Schoenfeld, U.C. Berkeley

Bob Floden, Michigan State University


Robust Algebraic Understandings
What skills and understandings do we (think we) want students to develop, to demonstrate a *robust understanding* of (to be effective at dealing with) verbally presented, situationally-based problems resolvable by algebra?
Classroom Analyses
How do we "capture" the classroom practices that we believe lead to students' robust understandings?
Pre- and Post-Tests
How do we "capture" the presence or absence of students' robust understandings?

The Big Question for today:

Classroom Analyses

How do we “capture” the classroom practices that we believe lead to students’ robust understandings?

Part 2

Trying to figure out "what counts"

Videos I'd show, if I had time –

to illustrate the range of contexts
we need to be able to code…

Sample (possible) Videos:

- The TIMSS U.S. Geometry lesson
- A highly regarded algebra teacher (MW) introducing students to work problems
- Another well regarded teacher making powerful use of Complex Instruction
- Imagine all the classrooms you know!

The TIMSS Geometry Video

This video exemplifies what we call IRE sequences:

Initiation (teacher asks a question)

Response (from the student)

Evaluation from the teacher.

A key feature: bite-size pieces of knowledge!

An Introduction to Algebra

The (highly regarded) teacher has given students the following problem to think about.

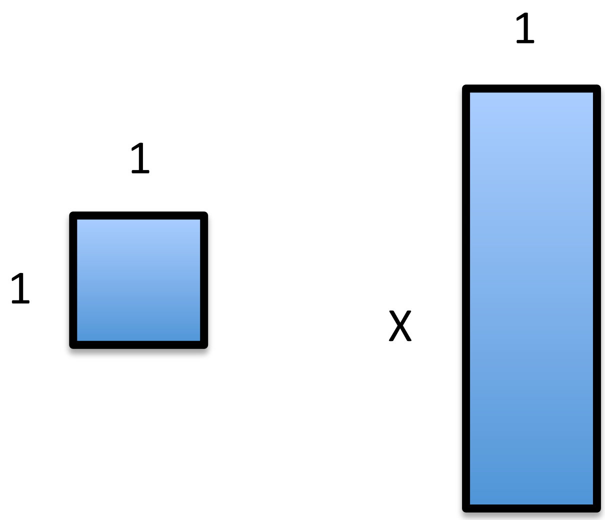
They have NOT seen problems like it:

"Carolyn can paint a fence in 6 hours. Georgia can paint the fence in 4 hours. How long will it take them to paint the fence together?"

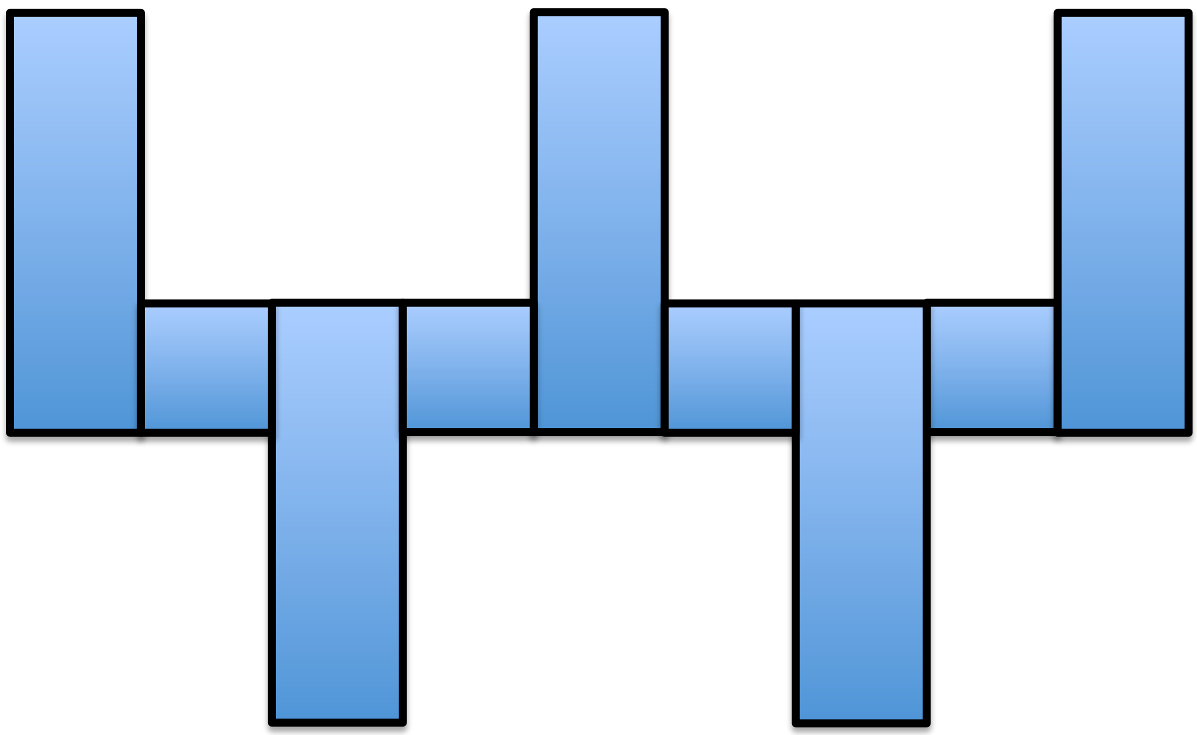
She fields suggestions, including some that don't seem to make sense, without evaluation.

Small Groups and Teacher Support (Complex Instruction)

The class is working on a problem, using these basic figures:



They're given the following task:
What is the perimeter of this figure?



Using Complex Instruction techniques, the teacher holds one student – and her table-mates – responsible for a complete and coherent explanation.

And now, think about the wide variety of classrooms you've visited.

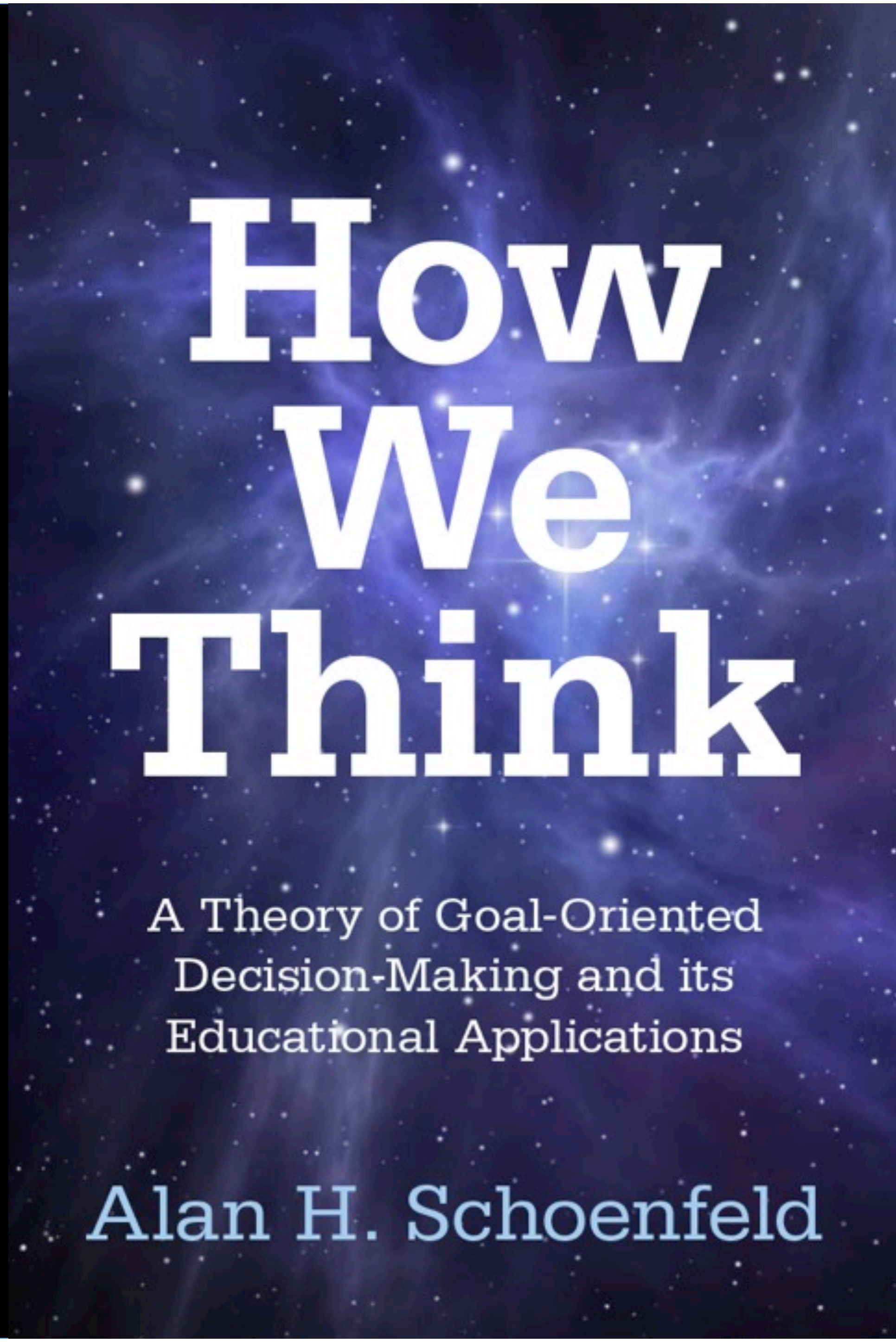
What really matters?

For a minute or two, work with those next to you, to identify categories of things you consider to be important – things that should be reflected in a coding scheme.

Part 3

The history behind our scheme

As many of you know, I've spent 25 years building and testing a theory of teachers' decision making:


How We Think
A Theory of Goal-Oriented Decision-Making and its Educational Applications
Alan H. Schoenfeld

Surely this theoretical scheme says what to look at in classrooms. Right?

Wrong.

After a year of trying, I gave up. It was WAY too complicated to parse lessons.

The literature will help, right?

After all, there's:

- Framework for Teaching (or FFT, developed by Charlotte Danielson of the Danielson Group),
- Classroom Assessment Scoring System (or CLASS , developed by Robert Pianta, Karen La Paro, and Bridget Hamre at the University of Virginia),
- Protocol for Language Arts Teaching Observations (or PLATO, developed by Pam Grossman at Stanford University),
- Mathematical Quality of Instruction (or MQI, developed by Heather Hill of Harvard University), and
- UTeach Teacher Observation Protocol (or UTOP, developed by Michael Marder and Candace Walkington at the University of Texas-Austin).
- IQA, Instructional Quality Assessment, developed by the University of Pittsburgh.

Actually, no.

- They all focus on important things, but they're all partial, or scattered, or have too many random parts; in some way or other none are close enough to use "off the shelf." They get at different things.
- So, we're back to building our own – while stealing as much as possible, of course.
- Here's our first try, in outline form.

	Access (what the teacher gives/allows)	Accountability (what the teacher expects/demands)	Productive Dispositions (what the teacher receives from students)
Strand	Dimensions (codes)	Dimensions (codes)	Dimensions (codes)
Mathematics	Students are able to experience the vibrancy and power of the domain of mathematics	Mathematical exploration and discussion should be accurate. Reasoning and justification should be tied to mathematics.	Students construct mathematics, attempting to discover rather than just receive.
Mathematics Learning	Students are given a chance to learn mathematics. This requires making making mathematics learning practices explicit and accessible.	Students are expected to engage productively in the mathematics learning process, sustain efforts, and contribute to finding solutions.	Students are interested in learning mathematics.
Classroom Community	No students are marginalized in the classroom community. All students have a chance to engage and participate.	Students have an obligation to their teacher and peers to be respectful and helpful. Students are not just participants but leaders of the classroom community.	Students contribute and participate as a community of mathematics practicioners.
Individual Learner	The classroom respects the uniqueness of each individual student, and gives appropriate affordances.	Students have an obligation to themselves to learn mathematics, and productively engage the subject matter.	Students sustain efforts as learners. Students take risks and believe that they can succeed.

We tried coding with this – YIKES!
Because the detail was MURDER:

Strand	Access (all students have opportunities to engage the subject)		Accountability (students are held to high standards)		Dispositions (student needs are met; students have productive dispositions)		Authority (students have ownership over their engagement with the subject)	
	Dimension	Constructs (codes)	Dimension	Constructs (Codes)	Dimension	Constructs (Codes)	Dimension	Constructs (Codes)
Mathematics	1-1. Access to rich mathematics	a) tasks provide opportunities to engage higher-level mathematical thinking b) the teacher presents tasks in a way that demand rich mathematical engagement	2-1. Accountable to the mathematics	a) teacher and students use multiple representations and make connections between representations; task requires multiple representations and connections between them. b) teacher presses for accuracy c) teacher asks probing questions/elicits reasoning and justification d) teacher and students use academic language e) teacher checks for understanding and provides feedback during instruction f) teacher builds on students' prior knowledge, connects mathematical ideas	3-1. Students view mathematics as:	a) a constructed body of knowledge b) useful	4-1. Authority over mathematical ideas	a) students generate/explain ideas b) students question, challenge, evaluate ideas
Mathematics Learning	1-2. Access to Explicit Expectations (taken from Ball's MQI)	a) teacher is explicit about what students have to do on a given problem b) teacher is explicit about how to use formal math language c) teacher is explicit about how to reason mathematically	2-2 Accountable to mathematics learning	a) teacher expects students to be able to learn mathematics b) teacher expect students to persist in mathematics learning	3-2. Students believe mathematics learning:	a) is achieved through hard work b) requires collaboration c) is rewarding/interesting	4-2. Authority to guide learning processes	a) students facilitate discussions b) students manage logistics c) students set the agenda/have choice in activities
Classroom Community	1-3. Opportunity to Receive (and Give) Meaningful, Constructive Feedback:	a) teacher provides feedback b) students give and receive feedback from other students	2-3. Accountable to classmates	a) discussion among students is math-focused c) teacher relates and connects student ideas to one another d) teacher revoices/marks student contributions e) students question and evaluate each other and teacher	3-3. Dispositions toward classmates	a) students show respect for each others' ideas	4-3. Authority is distributed appropriately throughout the class** **In our scheme, we should be careful to differentiate between normative and non-normative descriptors; it shouldn't look like the ideal is for students to have all the authority and teachers none, or vice versa.	a) across the teacher and the students* b) between pre-existing ideas and ideas generated by the class* *captured by three kinds of "who" in codes cited above: 1) teacher, 2) students, and 3) explicit teacher support for students to engage in X (some codes also imply the additional "who" of outside authorities, such as textbooks or some "They" that might make the rules)
Individual Learner	1-4. Opportunity to Engage the Mathematics in Their Own Way.	a) teacher permits use of non-dominant language b) students engage the mathematics on their own level c) teacher provides students time to work independently d) tasks have multiple entry points e) problem contexts respect students' cultural backgrounds/prior knowledge	2-4. Accountable to themselves	a) students have a role as mathematical authorities b) students sustain efforts to reach learning goals c) students participate in classroom activities	3-4. Students feel:	a) like individuals capable of learning math b) it's okay to make mistakes c) like they have a mathematical future - from Davis & Seashore rubric	4-4. Students acquire authority through competence.	a) teacher positions students as competent b) teacher positions students as *capable* of doing the math - from Ball's MQI and Cohen's complex instruction

Code #	Feasible in Real-time?	Focus Area	Time Scale	Spatial Scale	Description of Code	ACCESS	ACCOUNTABILITY	DISPOSITIONS	AUTHORITY
TEACHER									
1T	Y	Teacher	Lesson	Whole Class	When setting up a task, teacher checks whether students understand the directions	1-a: Explicit Expectations about - what to do on a given task		3-1. Teacher responds to students' disposition toward mathematics as	
2T	Y	Teacher	Lesson	Whole Class	Teacher checks for understanding. (an absolute count of number of times we observe this, either formally through unit assessments, or informally through quick-and-dirty formative in-class quizzes or even small-group questioning)	4-a: Opportunity to Receive Feedback - from the teacher		3-1. Teacher responds to students' disposition toward mathematics as	
3T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher pushes for conceptual understanding (e.g., through "Why?" questions) - (absolute count)	To rich mathematics? (No construct yet about this)	1: Accountability to the Math	3-1. Teacher responds to students' disposition toward mathematics as	4-a: Students positioned as competent (which gives them authority)
4T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher asks students to justify/explain their reasoning. [Is this an example of 3T?]	1-c: Explicit Expectations - about how to reason in math	1: Accountability to the Math		
5T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher prompts students to respond to each other's ideas (absolute count)	4-b: Opportunity to Receive Feedback - from other students	4: Accountability to Classmates	3-2 Teacher responds to students' disposition toward mathematics learning	1-b: Authority to - question, challenge, evaluate math ideas
6T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher solicits student ideas.	3-b: Access to Productive Identities - students positioned as capable learners		3-2 Teacher responds to students' disposition toward mathematics learning	1-a: Authority to - generate/explain math ideas
7T	Y	Teacher	Lesson	Whole Class	Teacher takes up or ignores a student idea. [How does it work to have both "taking up" and "ignoring" as the same code? -NLL]	-		3-2 Teacher responds to students' disposition toward mathematics learning	1-a: Authority to - generate/explain math ideas
8T	Y	Teacher	Lesson	Whole Class / Small Group	Teacher builds on students' prior mathematical knowledge [I need an example here more than on the others; what would this look like? -NLL]	2-b: Engaging the Math in Own Way - on their own math level			
	?	Teacher	Lesson / Unit	Whole Class	Teacher pushes students toward mathematical accuracy and toward formal math terminology [maybe examples would be, "teacher explicitly teaches mathematical language and vocabulary," and/or "teacher revoices student ideas in formal mathematical language." I think is is feasible to code these in real time. -NLL]	1-b: Explicit Expectations about - using formal math terminology	2-1: Accountability to the Math		
	?	Teacher	Unit	-	Teacher makes future-oriented statements about kids using or doing math in the future in some way (Davis & Seashore have a 4-point rubric in their scheme we can look at; We also could make a tally of the number of such statements that occur over the course of a unit; Or, we could just tally yes/no per lesson and then analyze the pattern over the course of the unit)	3-a: Access to Productive Identities - envisioning a mathematical future			
	?	Teacher	Unit	-	Teacher makes an encouraging remark that may, for example, foster persistence or position students as capable learners (We could make a tally of the number of such statements that occur over the course of a unit)	3-b: Access to Productive Identities - students positioned as capable learners		4-a: Students see themselves as capable	1-a: Authority to explain/generate math ideas 4-a: Students positioned as competent (which gives them authority)
	N	Teacher	Lesson	Whole Class / Small Group	Wait time. (calculate the average time a teacher waits for student response after asking a question)	2-c: Engaging the Math in Own Way - students have independent work/think time			
STUDENTS									
4S	Y	Students	Lesson	Whole Class / Small Group	Students justify/explain their reasoning.	-	1: Accountability to the Math		1-a: Authority to - generate/explain math ideas
5S	Y	Students	Lesson	Whole Class / Small Group	Students question and evaluate mathematical ideas, whether they come from the teacher or from classmates. (an absolute count - this may happen in whole group discussion or small-group work)	4-b: Opportunity to Receive Feedback - from other students			1-b: Authority to - question, challenge, evaluate math ideas
6S	Y	Students	Lesson	Whole Class / Small Group	Students share new ideas.	-		3-3. Students dispositions toward classroom community (classmates or the teacher)	1-a: Authority to - generate/explain math ideas
9S	Y	Students	Unit	Whole Class / Small Group	Students facilitate whole-class or small group discussions (yes/no)	-		3-3. Students dispositions toward classroom community (classmates or the teacher)	2-a: Authority over classroom activity - facilitating discussions
10S	Y	Students	Lesson / Unit	Whole Class	Students are responsible for logistical tasks (e.g., passing out papers) - (yes/no)	-		3-4. Students dispositions toward individual/self-efficacy	2-b: Authority over classroom activity - managing logistics
11S	Y	Students	Lesson	Whole Class / Small Group	Especially in classes with ELL students, students are observed using non-dominant language in class without sanction from teacher (yes/no).	2-a: Engaging the Math in Own Way - through use of non-dominant language		3-3. Students dispositions toward classroom community (classmates or the teacher)	4-a: Students positioned as competent (which gives them authority)
12S	Y	Students	Lesson	Whole Class / Small Group	Participation is distributed fairly across students so that no handful of students dominate discussion	2-c: Engaging the Math in Own Way - students have independent work/think time		3-3. Students dispositions toward classroom community (classmates or the teacher)	
	N	Students	Lesson	-	% of time students spend working on math independently (compared with time spent on teacher talking about math or classroom management)	2-c: Engaging the Math in Own Way - students have independent work/think time		3-4. Students dispositions toward individual/self-efficacy	
	?	Students	Lesson / Unit	Whole Class	Students participate in setting lesson agenda and structuring activities (e.g., who to work with, how much time spent on an activity etc.) - (yes/no)	-		3-1. Teacher responds to students' disposition toward mathematics as	2-c: Authority over classroom activity - setting lesson agenda
TASK									
4K	Y	Task	Lesson	-	Task requires students to justify, conjecture, interpret		1: Accountability to the Math	3-1: Nature of mathematics	
	N	Task	Lesson	-	Task affords multiple entry points for students.	2-b: Engaging the Math in Own Way - on their own math level		3-1: Nature of mathematics	4-4b: Students are positioned as competent/capable
	N	Task	Lesson	-	Task affords multiple representations	2-b: Engaging the Math in Own Way - on their own math level	1: Accountability to the Math	3-1: Nature of mathematics	
	?	Task	Lesson	-	Tasks have real-world applications			3-1: Nature of mathematics	

There were codes
For teacher, students,
And task along
All the dimensions.
Bleh!

EoI's

- We tried again, looking for "Events of Interest," or EoI's. What can we say when something important or interesting happens?
- There were 3 sets of EoI's (classroom climate, mathematical norms, and algebra specifics), and corresponding rubrics:

ACTION 2.0		Algebra Classroom Teacing Instrument for Observing Norms
Events of Interest		
Part 1: Classroom Context	**Event #**	**Description of Event**
A. Lesson Goal	1	Teacher explicitly states lesson goals
	2	Teacher writes down lesson goals
	3	Time not spent on achieving lesson goals (tally time spent on administrative, or discipline issues, mathematics that does not relate to lesson goals.)
B. Processes	4	Teacher explicitly specifies the product
	5	Teacher provides guidelines on how to work on the task (small group, individual, etc)
	6	Teacher specifies amount of time alloted to work on task
	7	Teacher states expected qualitites of work (see IQA)
C. Classroom Climate	8	Teacher manages behavioral disruptions
	9	Students participate in small group work (see rubric)
	10	Students participate in discussion (see rubric and IQA)
D. Task as Written	11	The task requires students to (1) navigate the language, (2) identify and relate relevant quantities, (3) Represent quantities (4) Solve problem, (5) Explain reasoning

Events of Interest		
Part 2: General Mathematics	**Event #**	**Description of Event**
A. Big Ideas/ Mathematical robustness	1	Teacher highlights a mathematically central idea (how and why it works).
	2	Teacher makes a superficial/trivial attempt to highlight a mathematical idea.
B. Mathematical Accuracy	3	Teacher makes a significant mathematical error.
	4	Teacher makes a minor mathematical error.
C. Scaffolding	5	Teacher provides scaffolding that helps students who are stuck without compromising the mathematics.
	6	Teacher trivializes the task by providing an explicit procedure.
D1. Teacher presses for student reasoning	7	Teacher presses for accuracy or asks students to provide evidence for claims.
	8	Teacher makes a superficial/formulaic attempt to ask students to provide evidence.
D2. Students explain and press for explanations	9	Student provides appropriate evidence for a claim.
	10	Student provides superficial evidence for a claim.
E. Use of student ideas (a.k.a. formative assessment	11	Teacher elicits student ideas and pursues correct reasoning to deepen understanding, or incorrect reasoning to help correct misunderstandings.
	12	Teacher makes a superficial/trivial attempt to elicit student ideas, but does not productively use them.

Events of Interest			
Part 3: CAT-specific Events	Sub-Category	Event #	Description of Event
A. Navigating Language		1	Participants rephrase/reword the problem context to put it in more kid-friendly language.
		2	Teacher checks that students understand non-mathematical vocabulary.
		3	Teacher checks that students understand mathematical vocabulary.
		4	Evie: use of reading strategies, students being asked to read aloud or in small groups, word walls, use of personal dictionaries, sentence frames, sentence starters
B. Identifying Relevant Quantities		5	Teacher asks questions that call students attention to relevant quantities (e.g., What is the problem asking you to find? or What does the problem give you?)
		6	Evie: Students connect quantities, operations, relationships, and calculations to reasoning around context.
		7	Evie: Students make sense of the quantities required to solve the problem.
		8	Evie: Students articulate goals or strategies for solving problem connected to reasoning around context.
C. Representing Relevant Quantities	C-1. Articulating Mathematical Relationships Between Quantities	9	Participants make explicit connections between inputs and outputs (vs. relying on recursive rules).
		10	Participants engage in qualitative sense-making of relationships between quantities.
		11	Participants reference a family/families of functions and their features.
	C-2. Generating Representations	12	Kim: Students choose which representation to use
		13	Kim/Dan: Students construct a representation (e.g., equation, graph, table).
		14	Bob: Teacher asks the students to construct a representation / The task requires students to construct a representation.
		15	Alan: The representation is tied in a meaningful or useful way to the context of the problem.
	C-3. Interpreting or Making Connections Between Representations	16	Participants move between representations.
		17	Participants use representations to solve contextual problems.
		18	Participants compare the advantages and/or limitations of various representations.
		19	Evie: participants make connections among representations (it's not just comparing representations, like "I like the table better than a graph"; it's about seeing how the rate of change, for example, shows up in the table and in the graph)
D. Solving the Problem	D-1. Making Calculations or Executing Procedures	20	Bob: Teacher emphasizes arithmetical accuracy or providing opportunities for students to do calculations correctly (providing resources, etc.)
		21	Participants solve an equation for a variable.
		22	Participants use algebraic techniques to solve systems of equations (substitution, elimination, etc. vs. guess-and-check)
	D-2. Attending to the Problem Context to Check the Plausibility of Results or Making Sense of Quantities	23	Participants orally reference the problem context in explaining their work Or Participants reference the problem context in explaining their work in writing.
E. Justifying and Explaining Reasoning		24	?????
		25	?????

Yikes! This was way too complex also.

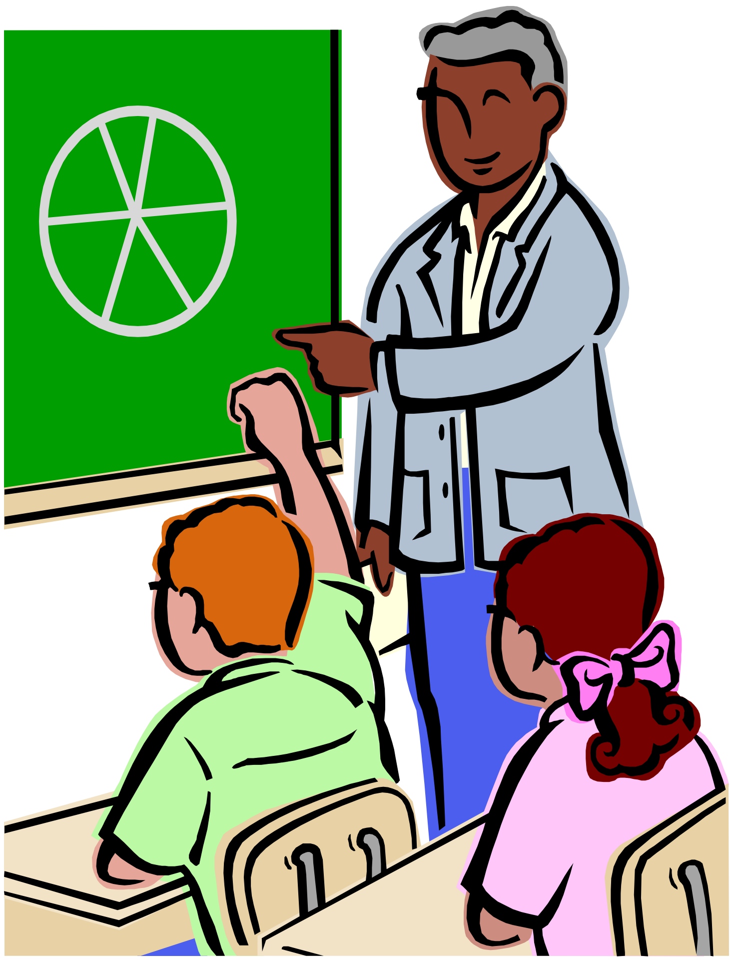
I'll skip the next iteration

(Say Thanks!)

Version 4: A new Approach (with elements of the old):

- First, parse the lesson into episodes.
- Then, look at salient aspects of each episode ("facets," e.g., the way the math is treated or the way kids are set up to work).
- Then, for each facet, score the relevant dimensions of activity.

Step 1: Observe & Take Field Notes



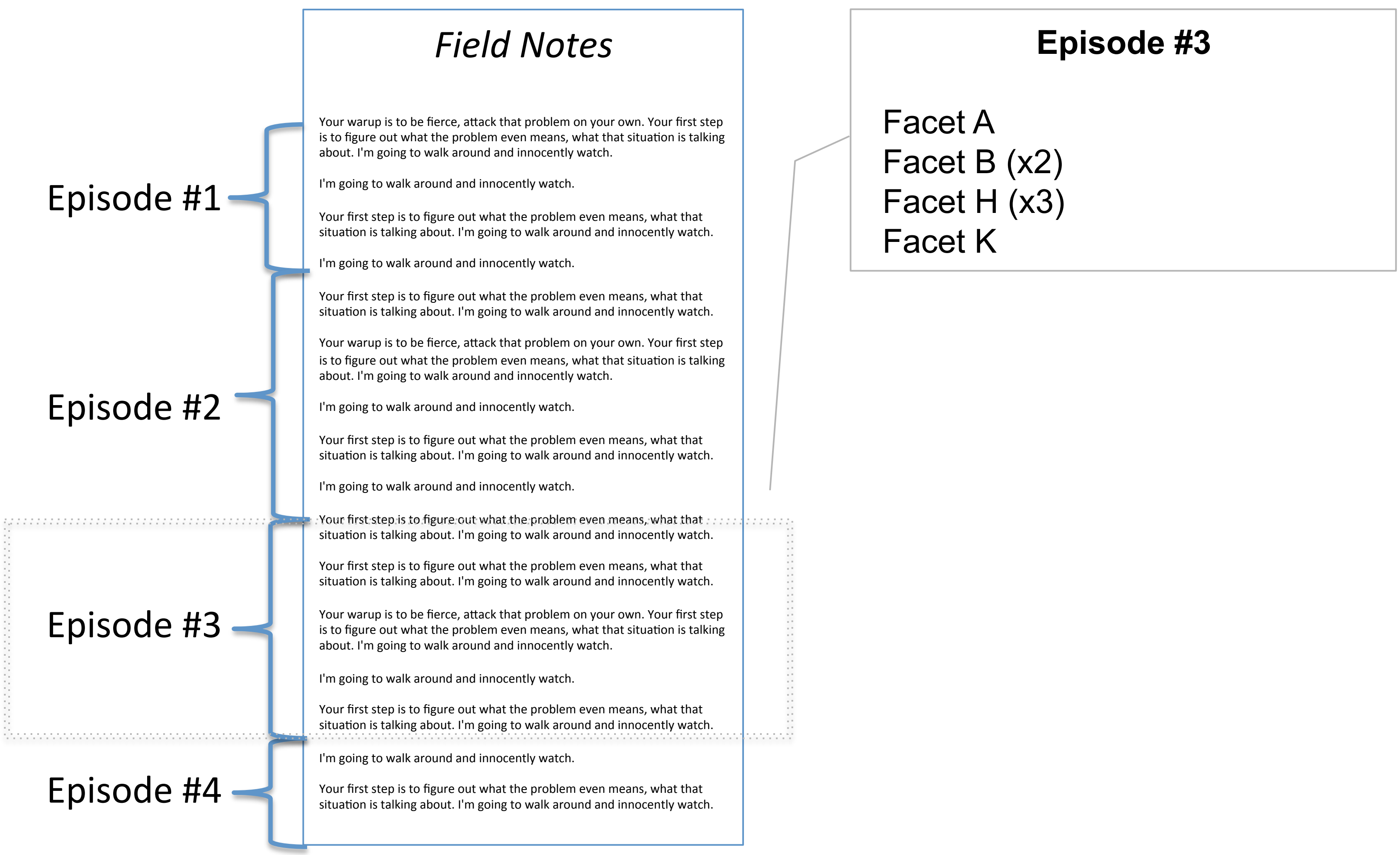
Step 2: Chunk Your Field Notes into Episodes

An episode ends when…

- When a new mathematical idea/topic is being discussed
- When the class moves on to another part of a task
- When the class switches from whole class <-> small group

45 sec < episode < 5 min

Step 3: Identify the Facets of Each Episode

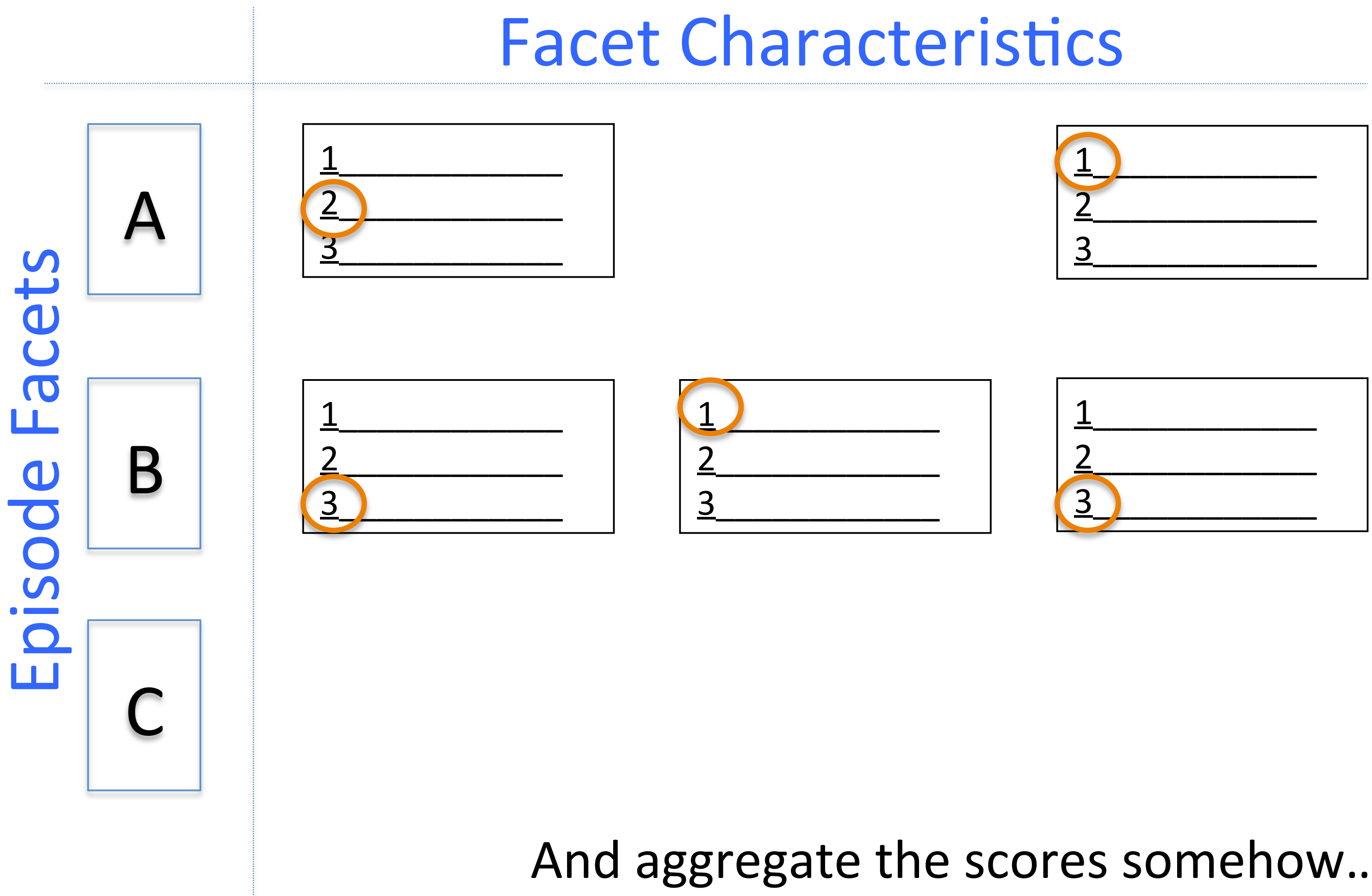


Each facet has various characteristics, e.g.,

ACTION 4.1 2011-11-3

#	Facet			
A	Giving Directions (for Individual or Group Work)	*Setting Process Expectations*	* Setting Product Expectations*	
		1 Teacher tells students to get started without setting process expectations. 2 Teacher sets process expectations (e.g., amount of time for task, how students should organize themselves). 3 Teacher engages students in mutually setting process expectations.	1 Teacher tells students to get started without setting product expectations. 2 Teacher sets expectations about final product (e.g., by providing a scoring rubric, showing examples of high quality work). 3 Teacher engages students in mutually setting expectations for final product.	
B	Summarizing the Math Discussed	Who is Doing the Summarizing?	What is the Nature of the Math Being Summarized?	
		1 2 3	1 2 3	
C	Connecting to Prior Knowledge	Who is Involved in Creating the Connections to Prior Knowledge?	What is the Nature of the Math Being Connected?	
		1 2 3	1 2 3	
D	Positioning Students Relative to Task	Who is Being Positioned as Capable of Doing the Math?	How/Why is the Math Being Learned Relevant/Useful?	What Does it Take to Be Successful in Math?
		1 Teacher tells students to work on task but doesn't position them relative to the task. 2 Teacher positions students as capable of working on a difficult task, but addresses students in a general way (e.g., you guys can do this). 3 Teacher is explicit in positioning ALL students as capable of working on the task (e.g., multiple ability treatment).	1 Mathematics is not emphasized as important/relevant to students. 2 Teacher talks about the importance of mathematics for students in a general sense (e.g., you guys really need to know this). 3 Utility of math is addressed specifically (e.g. students are positioned as having mathematical futures).	1 Teacher doesn't emphasize effort over ability. 2 Teacher emphasizes the importance of effort. 3 Teacher emphasizes the importance of effort AND the need to be persistent in the face of difficulty.

Step 4: Score Each Facet's Characteristics



And aggregate the scores somehow…

This was nice(r) in theory, but…

A MESS in implementation. Too many things to keep track of!

…because the facets I showed you represented only the tip of the iceberg…

ACTION 4.1 2011-11-3

#	Facet				
A	Giving Directions (for Individual or Group Work)	*Setting Process Expectations*	* Setting Product Expectations*		
		1 Teacher tells students to get started without setting process expectations.	1 Teacher tells students to get started without setting product expectations.		
		2 Teacher sets process expectations (e.g., amount of time for task, how students should organize themselves).	2 Teacher sets expectations about final product (e.g., by providing a scoring rubric, showing examples of high quality work).		
		3 Teacher engages students in mutually setting process expectations.	3 Teacher engages students in mutually setting expectations for final product.		
B	Summarizing the Math Discussed	Who is Doing the Summarizing?	What is the Nature of the Math Being Summarized?		
		1	1		
		2	2		
		3	3		
C	Connecting to Prior Knowledge	Who is Involved in Creating the Connections to Prior Knowledge?	What is the Nature of the Math Being Connected?		
		1	1		
		2	2		
		3	3		
D	Positioning Students Relative to Task	Who is Being Positioned as Capable of Doing the Math?	How/Why is the Math Being Learned Relevant/Useful?	What Does it Take to Be Successful in Math?	
		1 Teacher tells students to work on task but doesn't position them relative to the task.	1 Mathematics is not emphasized as important/relevant to students.	1 Teacher doesn't emphasize effort over ability.	
		2 Teacher positions students as capable of working on a difficult task, but addresses students in a general way (e.g., you guys can do this).	2 Teacher talks about the importance of mathematics for students in a general sense (e.g., you guys really need to know this).	2 Teacher emphasizes the importance of effort.	
		3 Teacher is explicit in positioning ALL students as capable of working on the task (e.g., multiple ability treatment).	3 Utility of math is addressed specifically (e.g. students are positioned as having mathematical futures).	3 Teacher emphasizes the importance of effort AND the need to be persistent in the face of difficulty.	
E	Teacher Exposition of Mathematical Ideas	[Incorporating Ideas from Class Discussion into Exposition]	[Depth/Quality of the Math in the Exposition]		
		1 Teacher ignores or dismisses student reasoning.	1		
		2 Teacher acknowledges contribution but doesn't actively incorporate it into the lesson (e.g., that's an interesting idea, but we're not working on that now).	2		
		3 Teacher incorporates and builds on student reasoning to move the lesson forward	3		
F	Discussing Mathematical Ideas/Reasoning	[Facilitating Discussion Participants]	[Eliciting Student Reasoning]	[How Student Responses are Taken Up]	[Encouraging Multiple Solution Paths]
		1 Only the first student that raises his/her hand is the one that gets called on.	1 Teacher does not attempt to further explicate student's thinking.	1	1 The task/introduction strongly suggests a single solution path.
		2 Beyond the first student, at least one other student who raised his/her hand gets called on to respond to a given question.	2 Teacher attempts to explain/re-phrase the students' thinking.	2	2 The task/introduction affords multiple potential solution paths.
		3 Teacher uses techniques to actively engage students who do not volunteer (e.g., wait time, popsicle sticks, cold calling).	3 Teacher probes student to further explicate his/her strategy/thinking.	3	3 The task/introduction encourages/requires multiple solution paths and/or the contrast of different solutions.
G	Monitoring Whole Class Understanding - INFORMAL	How Deep was the Math Being Assessed?	How Many Students are We Getting Data From?	What Does the Teacher Do with This Information?	
		1 The monitoring only involved checking answers (i.e., "How many of you got 3/4 for #17?")	1	1	
		2 The monitoring had to do with assessing students' execution of a mathematical procedure.	2	2	
		3 The monitoring asked students to explain their reasoning or answer a why question.	3	3	
H	Monitoring Whole Class Understanding - FORMAL	How Deep was the Math Being Assessed?	How Many Students are We Getting Data From?		
		1 The monitoring only involved checking answers (i.e., "How many of you got 3/4 for #17?")	1		
		2 The monitoring had to do with assessing students' execution of a mathematical procedure.	2		
		3 The monitoring asked students to explain their reasoning or answer a why question.	3		
I	Student Seeks to Clarify Mathematical Ideas/Reveals Confusion	How Cognitively Demanding is the Response?	How Cognitively Demanding is the Student's Question?	How is the Question Taken Up?	
		1 Teacher ignores or dismisses the question.	1 The student asks about whether an answer is correct or not (i.e., a "WHAT" question) or a non-specific question (e.g., "I don't know how to get started!")	1 Acknowledged but not responded to.	
		2 Teacher gives an explanation directly answering the student's question.	2 The student asks a specific question about HOW to do a procedure.	2 Answered by teacher.	
		3 Teacher engages the student/class in answering the question (e.g., acting as a guide).	3 The student asks WHY something works.	3 Students engaged in answering it.	
J	Scaffolding the Mathematics in the Tasks	[Maintaining Cognitive Demand]	[Providing a Variety of Entry Points]		
		1	1		
		2	2		
		3	3		

…so, we needed to fix things again…

But, we thought this had the right stuff. Just scrambled.

So, how about creating equivalence classes:

(1) important types of classroom situations.

(2) important dimensions of the lesson, which we would examine in those situations.

Et voilà: a scheme that's actually workable

(we think.)

Let's do situations first. Here are important situations to look at:

- Whole Class Discussions
- Small Group work
- Student Presentations
- Individual work

And here's what to look at in them:

five dimensions of classroom interactions.

Key Questions for Math Classes:

- Was there honest-to-goodness math in what students and teacher did?
- Did students engage in “productive struggle,” or was the math dumbed down to the point where they didn’t?
- Who had the opportunity to engage? A select few, or everyone?
- Who had a voice? Did students get to say things, develop ownership?
- Did instruction find out what students know, and build on it?

Was there honest-to-goodness math in what students and teacher did?

Level	Mathematical Focus, Coherence and Accuracy
1	Classroom activities are purely rote, **OR** disconnected or unfocused, **OR** consequential mistakes are left unaddressed.
2	The mathematics discussed is relatively clear and correct, **BUT** connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.
3	The mathematics discussed is relatively clear and correct, **AND** connections between procedures, concepts and contexts (where appropriate) are addressed and explained.
	Mathematical Focus, Coherence and Accuracy

Did students engage in "productive struggle," or was the math dumbed down to the point where they didn't?

Level	**Cognitive Demand**
1	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.
2	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.
3	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.
	Cognitive Demand

Who had the opportunity to engage? A select few, or everyone?

Level	Access
1	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.
2	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.
3	The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.
	Access

Who had a voice? Did students get to say things, develop ownership?

Level	Agency: Authority and Accountability
1	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.
2	Students have a chance to say or explain things, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.
3	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each others' ideas.
	Agency: Authority and Accountability

Did instruction find out what students know, and build on it?

Level	Uses of Assessment
1	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.
	Uses of Assessment

Put everything together with these as the dimensions:

Level	Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
1	Classroom activities are purely rote, **OR** disconnected or unfocused, **OR** consequential mistakes are left unaddressed.	Classroom activities are structured so that students mostly apply familiar procedures or memorized facts.	Classroom management is problematic to the point where the lesson is disrupted, **OR** a significant number of students appear disengaged and there are no overt mechanisms to support engagement.	The teacher initiates conversations. Students' speech turns are short (one sentence or less) and shaped or constrained by what the teacher says or does.	The teacher may note student answers or work, but student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The mathematics discussed is relatively clear and correct, **BUT** connections between procedures, concepts and contexts (where appropriate) are either cursory or lacking.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges and mostly limit students to providing short responses to teacher prompts.	The class is engaged in mathematical activity, but there is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Students have a chance to say or explain things, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The mathematics discussed is relatively clear and correct, **AND** connections between procedures, concepts and contexts (where appropriate) are addressed and explained.	The teacher's hints or scaffolds support students in "productive struggle" in building understandings and engaging in mathematical practices.	The teacher actively supports (and to some degree achieves) broad and meaningful participation, **OR** what appear to be established participation structures result in such participation.	Students put forth and defend their ideas. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each others' ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.
	Mathematical Focus, Coherence and Accuracy	**Cognitive Demand**	**Access**	**Agency: Authority and Accountability**	**Uses of Assessment**

and the situations I mentioned before, and you get…

The Teaching for Robust Understanding of Math (TRU Math) Scheme

Episode Type		Mathematical Focus, Coherence and Accuracy	Cognitive Demand	Access	Agency: Authority and Accountability	Uses of Assessment
Whole Class Activities: **L**aunch, Teacher **E**xposition, Whole Class **D**iscussion		How accurate, coherent, and well-justified is the mathematical content?	To what extent are students supported in grappling with and making sense of mathematical concepts?	To what extent does the teacher support access to meaningful participation for all students?	To what degree are students the source of ideas and discussion of them? How are student contributions framed?	To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas (when potentially valuable) or address misunderstandings when they arise?
	1					
	2					
	3					
Small **G**roup work		How accurate, coherent, and well-justified is the mathematical content?	To what extent does teacher support students to interact with their group members to make sense of mathematical concepts, or individuals by themselves?	To what extent does teacher support/ group dynamics provide access to meaningful participation and "voice" for all students?	To what extent does teacher support/ group dynamics provide access to meaningful participation and "voice" for all students?	To what degree does the teacher monitor and help students refine their thinking within small groups?
	1					
	2					
	3					
Student presentations		How accurate, coherent, and well-justified is the mathematical content?	Cognitive Demand	Access	To what degree are students the source of presented ideas and response to presented ideas?	To what degree does the teacher use student presentations to support meaningful class engagement with core ideas?
	1					
	2					
	3					
Individual work		How accurate, coherent, and well-justified is the mathematical content?	To what extent are students supported in grappling with and making sense of mathematical concepts?	Does the teacher encourage meaningful engagement from all students?	How does the teacher frame individual student contributions? To what degree do students get to propose and defend their own ideas?	To what degree does the teacher explore student thinking about a problem (whether right or wrong) and work with the student on it?
	1					
	2					
	3					

We think this is a workable general scheme, which captures what we think is important in math classes.

But remember, our work has an algebra-specific focus, as well as a general math component:

Events of Interest			
Part 3: CAT-specific Events	Sub-Category	Event #	Description of Event
A. Navigating Language		1	Participants rephrase/reword the problem context to put it in more kid-friendly language.
		2	Teacher checks that students understand non-mathematical vocabulary.
		3	Teacher checks that students understand mathematical vocabulary.
		4	Evie: use of reading strategies, students being asked to read aloud or in small groups, word walls, use of personal dictionaries, sentence frames, sentence starters
B. Identifying Relevant Quantities		5	Teacher asks questions that call students attention to relevant quantities (e.g., What is the problem asking you to find? or What does the problem give you?)
		6	Evie: Students connect quantities, operations, relationships, and calculations to reasoning around context.
		7	Evie: Students make sense of the quantities required to solve the problem.
		8	Evie: Students articulate goals or strategies for solving problem connected to reasoning around context.
C. Representing Relevant Quantities	C-1. Articulating Mathematical Relationships Between Quantities	9	Participants make explicit connections between inputs and outputs (vs. relying on recursive rules).
		10	Participants engage in qualitative sense-making of relationships between quantities.
		11	Participants reference a family/families of functions and their features.
	C-2. Generating Representations	12	Kim: Students choose which representation to use
		13	Kim/Dan: Students construct a representation (e.g., equation, graph, table).
		14	Bob: Teacher asks the students to construct a representation / The task requires students to construct a representation.
		15	Alan: The representation is tied in a meaningful or useful way to the context of the problem.
	C-3. Interpreting or Making Connections Between Representations	16	Participants move between representations.
		17	Participants use representations to solve contextual problems.
		18	Participants compare the advantages and/or limitations of various representations.
		19	Evie: participants make connections among representations (it's not just comparing representations, like "I like the table better than a graph"; it's about seeing how the rate of change, for example, shows up in the table and in the graph)
D. Solving the Problem	D-1. Making Calculations or Executing Procedures	20	Bob: Teacher emphasizes arithmetical accuracy or providing opportunities for students to do calculations correctly (providing resources, etc.)
		21	Participants solve an equation for a variable.
		22	Participants use algebraic techniques to solve systems of equations (substitution, elimination, etc. vs. guess-and-check)
	D-2. Attending to the Problem Context to Check the Plausibility of Results or Making Sense of Quantities	23	Participants orally reference the problem context in explaining their work Or Participants reference the problem context in explaining their work in writing.
E. Justifying and Explaining Reasoning		24	?????
		25	?????

The algebra-specific part remains, in the form of our "robustness criteria":

RC1	Reading and interpreting text, and understanding the contexts described in problem statements.
RC2	Identifying salient quantities in a problem and articulating relationships between them
RC3A	Generating representations of relationships between quantities
RC3B	Interpreting and making connections between representations
RC4A	Executing calculations and procedures with precision
RC4B	Checking plausibility of results
RC5A	Opportunities for Student Explanations
RC5B	Teacher instruction about Explanations
RC5C	Student Explanations and Justifications in Whole-Class Discussion

We also code for these, but there isn't time to go into detail.

Part 4

Using the scheme to code a video

Coding session

- Watch the video of the "Border Problem," Part 1
- 5 minutes to code the dimensions, by yourself
- 5 minutes to discuss codings with partners

  During those 10 minutes, note issues.
- Collective discussion of codings.

Part 5

Discussion.

The floor's open!

Part 6

Reflections on PD

What do you need for successful PD?

- A (theoretically grounded) vision
- Systemic Coherence
- Tools
- Mechanisms for building community and supporting teachers

A (theoretically grounded) vision

Well, we just talked about that!

Systemic coherence:

- In general, you need all of these to be aligned:

- standards, curriculum, assessment, PD; and enough time and stability for things to take hold.

- For Internal Coherence in PD: **Everyone** experiences math lessons the way they should be taught.

Tools And Community Building:

Tools:

“Formative Assessment Lessons” – google “Mathematics Assessment Project” – and the TRU Math Scheme

Community Building:

Lesson study using FALs and TRU Math.

Theory of Action for The Final Mile