Highlighting heritages and building tasks:

A CRITICAL ANALYSIS OF LITERATURE ON MATHEMATICS CLASSROOM DISCOURSE

Beth Herbel-Eisenmann

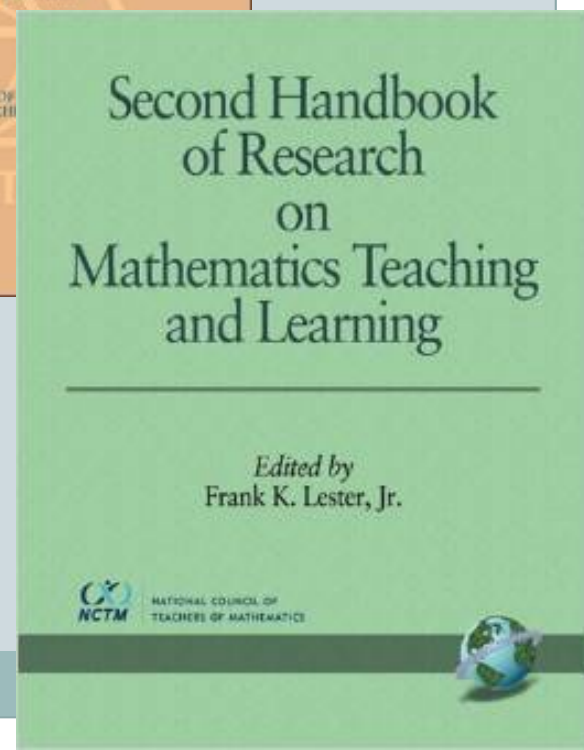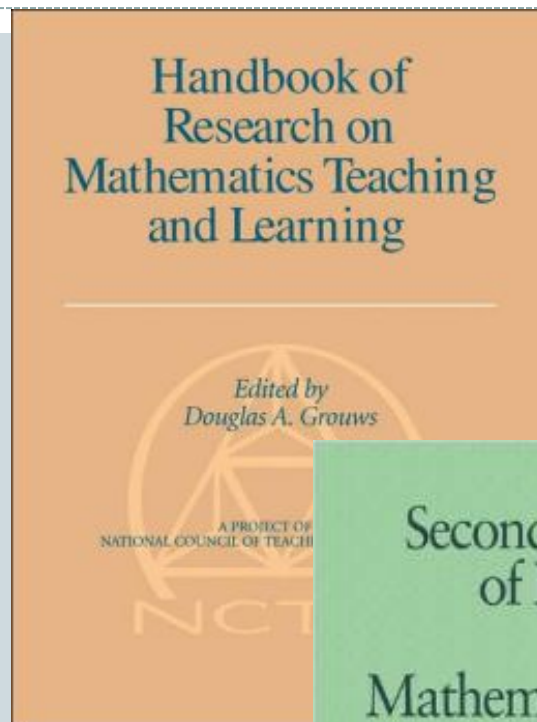Einat Heyd-Metzuyanim

Tamsin Meaney

Jessica Bishop

Background

# Handbook of Research on Mathematics Teaching & Learning

- First published in 1992 (Doug Grouws, Ed.)
- Second one published in 2007 (Frank Lester, Ed.)
- "New" handbook in 2017 (Jinfa Cai, Ed.)
- First time there will be a chapter on mathematics classroom discourse in this series

Handbook of
Research on
Mathematics Teaching
and Learning

Edited by
Douglas A. Grouws

A PROJECT OF
NATIONAL COUNCIL OF TEACH

NCTM

Second Handbook
of Research
on
Mathematics Teaching
and Learning

Edited by
Frank K. Lester, Jr.

NCTM NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Determining a "starting point"

- September 1974, *Interactions between Linguistics and Mathematical Education*, Nairobi, Kenya

- Austin and Howson (1979) on language & mathematics education; Lloyd Dawe (1983) Bilingualism & mathematical reasoning in *Educational Studies in Mathematics*

- Cuevas (1984); Carraher, Carraher & Schliemann (1987) in *Journal for Research in Mathematics Education*

- David Pimm's (1987) highly cited *Speaking Mathematically*
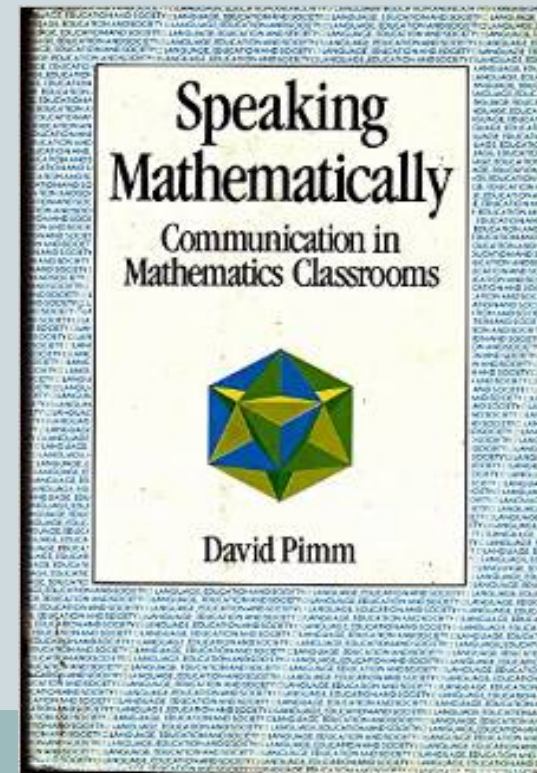
Criteria for Inclusion & Exclusion

Criteria for Inclusion

- Chose not to define "discourse" because of the wide range of meanings for this term in the field, instead we identified a set of ideas we consider to fall under a "discourse" umbrella

- Research focused on teachers, students, and textbooks/written texts in mathematics classrooms, K-16

- Primarily articles published in peer reviewed research journals

Criteria for Exclusion

- Professional development or teacher education

- Many books and edited books written for researchers and practitioners

- Previously published literature reviews about mathematics classroom discourse

- Gesture, embodiment & semiotic mediation, argumentation and proof, **bilingual & multilingual learners**, race & **identity** intersectionality, and affective dimensions of learning

Grouping for analysis

As we skimmed to determine inclusion, we noticed that there was great variation in the range of theoretical, conceptual, and methodological approaches authors used…

"Intellectual heritage" focus

- "Part A" (over 200 articles)
  - Some articles had more easily identified discursive heritages
- "Part B" (over 60 articles)
  - Some articles focused on mathematics classroom discourse but did not draw from discursive, linguistics, sociolinguistics, or discourse related intellectual heritages—tended to focus on "questioning" or "talk" but did not link to discursive theories or methods (e.g., used grounded theory)
  - Some articles drew on many different traditions, making it hard to discern the primary intellectual heritage

Part A: Discursive Heritages

- Cultural, social, and discursive psychology;
  - Articles drew from, e.g., Harré and van Langenhove, Lave and Wenger, Vygotsky, Wittgenstein
  - Took, as their starting point, some version of psychology
- Sociolinguistics and discourse studies;
  - Articles drew from, e.g., Austin & Searle, Blumer, Goffman, Grice, Halliday, Sacks
  - Took, as their starting point, some version of sociolinguistic or discourse perspective
- 'Reconceptualists'; and
  - We adopted this term from curriculum theorists—drew from critical, hermeneutic, psychoanalytic, post-structural, feminist, etc. perspectives
  - Articles drew from, e.g., Bernstein, Bourdieu, Derrida, Fairclough, Foucault, Gee, Gramsci, van Dijk, and Žižek
- 'Threads' of work
  - three or fewer articles were published in the 35 years of work
  - transactional writing approaches, psycholinguistics, and linguistics

Part B: Discursive Heritages

- Cultural, social, and discursive psychology;
  - Sociocultural theory; **Discursive psychology/ Commognition**; & **Positioning**
- Sociolinguistics and discourse studies;
  - Systemic functional linguistics; Emergent/Symbolic interactionism; Pragmatics [Gricean umbrella]; **Interactional sociolinguistics**
- 'Reconceptualists'; and
- 'Threads' of work

Analytic frameworks
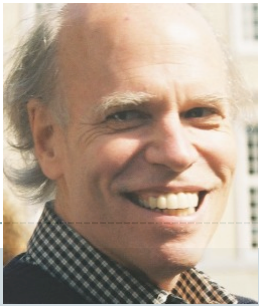
1st level of analysis

- Done to both Part A and Part B

Summary table 1

| Research? Focus/Goal	Define/ Conceptualize/Bound discourse	What motivates their study	Context	Object of Analysis	Qual/Quant & tools/ approaches	Key Findings

2ⁿᵈ level of analysis: Only Part A

Gee's (2011) "Building Tasks": *we use grammar and words to "build structures and their accompanying meaning" in order to "do things with language," such as think or perform actions* (p. 87).

- *Significance*
- *Activities*
- *Identities*
- *Relationships* (between people, but also between people and things like institutions)
- *Politics* (the distribution of social goods)
- *Connections* (between ideas and things)
- *Sign systems and knowledge*
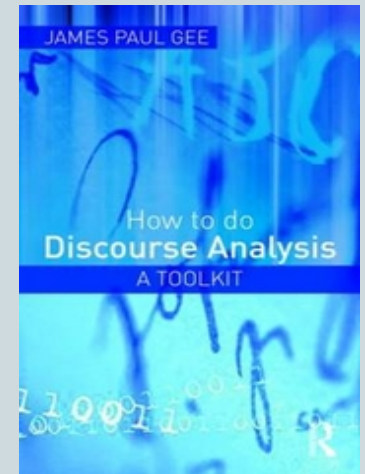
Significance	Activities	Identities	Relationships (at least one of the two involved in the relationship is a person/people)	Politics	Connections (between/ among ideas)	Sign Systems & Knowledge
What is/are the author trying to make a case for? What is the goal or point of the paper?	What kind of the activities does the author focus on? Examples: engaging in critical language awareness; the production of text	Does the author explicitly attend to identity or identities and in what ways? What does the researcher articulate or make explicit about him/herself in terms of positionality?	How does the author address relationships between and among students, teachers and institutions (mathematics, school, broader institutions)?	Does it address power explicitly? Do the authors explicitly question hegemonic practices?	Try to formulate the key concepts or ideas the author is focusing on and how they are related to each other. For example: voice-agency beliefs - practices	What does it mean to know and do mathematics?
What do the authors assume to be important? What key ideas get repeated over and over again?	What activity is the author engaging in as s/he does research and writes? (e.g., convince people of X; answer a research question with empirical evidence) What kind of research activity is valued? (e.g., how do they look at classroom discourse?)	What are the identities/ positionings/roles that the researcher takes on? What does the researcher's positionality statement tell us about his/her identities/ positioning/ roles?	What kind of relationships does the author promote?	In what ways does the author attend to the distribution of social goods (anything the society treats as a good worth having). (See Gee p. 90). What are the things they seem to value?	What kind of connections does the author promote? (e.g., the relationship between linguistic analysis and mathematics content)	What are the epistemological and philosophical assumptions about the relationship between language use (sign systems) and knowledge? What's the purpose of the research? (e.g. showing what children or people can do)

Second level of analysis: Only Part A

- Allowed us to identify what the articles in these heritages/sub-heritages foreground and background, in terms of understanding what people 'do with language' in mathematics classrooms
- Provides a mapping of the field
- Not about how *original* authors used these ideas, but about *how we are taking them up in mathematics education*

Three Heritages/Sub-Heritages

BROAD STROKES

Illustrations of Findings

- Positioning, n = 16
- Discursive psychology/Commognition, n = 25
- Interactional socio-linguistics, n=18

Positioning

Primarily draws on two lines of work

- Positioning theory (Harré & colleagues)
  - (speech act, storyline), and position/positioning as a mutually determining triad

- Theory of self & identity (Holland & colleagues)
  - Four contexts for production of personal and social identities-- figured worlds and negotiations of positionality (space of authoring and world making)

- Nine articles cite both

- Two articles cite neither (Gresalfi & Cobb, 2006; Hunter & Anthony, 2011) because focus is on "dispositions" (use "position" but do not define it)

Positioning literature

- Types of work
  - 4 theoretical/conceptual/methodological pieces
  - 12 empirical investigations
- Finds *Significance* in arguing for a broadening view of mathematics because of the ramifications for equity - want mathematics educators...
  - to consider learning mathematics as being about learning content & about learning to "be" particular kinds of people (that associate themselves with mathematics or not)
  - To see agency and authority as central to learning mathematics
  - Bring in ideas of funds of knowledge, culturally relevant practices, ethnomathematics, cultural backgrounds of students, history of communities
- Foregrounds *Political* and *Identity* building tasks (but only identity of youth, not authors)
  - "Power" and/or "empowerment" appear in every article

Bound/restrict 'discourse'

- Many focus on interaction, interaction & conversation, "language in use," participation structures, positioning acts or "discursive processes whereby selves are located in conversations as observably and subjectively coherent participants" (Davies & Harre, 1990)

- Discourse, when defined, is described as
  - Unfolding dramas (Wagner & Herbel-Eisenmann, 2009)
  - Spoken & written words, semiotic systems, representations, & gestures of participants as they use language to communicate, interact, or act (Bishop, 2012)
  - With figured worlds: "both capture socially organized and constructed ways of thinking, interacting, interpreting, and identifying" (Gresalfi, 2009; Esmonde & Langer-Osuna, 2012)

- How discourse is used
  - For meaning making (Bell & Pape, 2012)
  - As playing a "critical role in enacting identities" and the "primary method of shaping and conveying one's identity to others (Bishop, 2012)

Findings from positioning work show

- How mathematics classroom discourse structures authority relations in subtle, pervasive, and hegemonic ways (Herbel-Eisenmann & Wagner, 2010)

- When teacher's discourse moves open up possibilities for dialogic interaction, students can be positioned or position themselves to take up active, agentive roles (Bell & Pape, 2012; Hunter & Anthony, 2011)

- Small group work: problematic for some & empowering for others (Anderson, 2009; Bishop, 2012; Esmonde & Langer-Osuna, 2012; Kotsopolous, 2012; Langer-Osuna, 2011; 2014)

Other important aspects of this work

- Context is central-intimately related to how the interactions and positionings and figured worlds play out

- Much of it works across levels of context
  - Lexical bundles → broader issues of authority
  - Positioning → figured worlds of gender & romance and race
  - Analyses of micro-, meso-, and macro- levels to understand how individual acts of failing build up and "stick," resulting in some students being seen as "failures"

Discursive Psychology/ Commognition

Theoretical roots

- Vygotzky's theory of learning
- Wittgenstein's late philosophy; Harre & Gillete's (1994) "discursive psychology"
- Lave & Wenger (1991); Wenger (1998) "participationism"
- Main tenets:
  - Thinking is a form of communicating
  - Mathematics is a type of discourse
  - Discourses are defined by communities
  - Learning mathematics is a process of becoming a participant in a community

Scope and goals

- Mostly very detailed examinations of student-student and student-teacher interactions
  - In classrooms and out-of-classroom settings
  - Mathematical domains:
    - Early numerical reasoning (Sfard & Lavie, 2005)
    - Geometry (Sfard, 2007; Sinclair & Moss, 2012)
    - Algebra (Caspi & Sfard, 2012; Kieran, 2001; Nachlieli & Tabach, 2012; Sfard & Kieran, 2001),
    - Post-secondary education (Güçler, 2013; Kim et al., 2012, Ryve, 2004; 2006;).
- Goal of detailed examination: to uncover *mechanisms* of discursive development.

Building tasks in Commognitive works

- **Activity**: many of these papers engage in theoretical development and offer conceptual and methodological tools. For instance:
  - Offering terminology that does not imply dualism of thought and it's expression
    - Objectification ~ Conceptualization
    - Realization ~ Representation
    - Identity, identifying, subjectifying ~ beliefs, attitudes, self
  - Offering a theory of learning in socio-cultural context
    - From ritual to explorative participation

Building tasks in Commognitive works

- **Sign systems and knowledge**
  - Explicitly define mathematics as a type of discourse (human activity). Does not differentiate between knowledge and it's discursive production.
- **Identity**
  - A strand of this work deals explicitly with identity (Heyd-Metzuyanim & Sfard, 2012; Heyd-Metzuyanim, 2013; Heyd-Metzuyanim, 2015; Sfard & Prusak, 2005*) but most of it does not.
- **Politics**
  - Mostly does not deal explicitly with issues of power or equity.

Building tasks in commognitive works

- **Significance**:
  - Sfard's work: Stress the philosophical coherence of the theory. Arguments are put forward in terms of "logical deduction".
  - Attends to verbal as well as to non-verbal aspects of communication (with terms such as "visual mediators" and "focal analysis").

- **Relationships**:
  - Mostly attends to student-student and teacher-student relationships in formal (classroom) and informal learning contexts.

- **Connections**:
  - Connects between micro-analysis of classroom or interview talk and historical developments of mathematics.

Exemplary Findings

- One mechanism of discursive development: Commognitive conflict
  - A result of conflicts in meta-discursive rules (Sfard, 2007)
    - Different participants use the same words with different meanings
    - Incongruences often remain unnoticed
  - Kjeldsen & Blomhoj (2012): utility of studying history of mathematics for understanding meta-rules
  - Caspi & Sfard (2012): examined meta-level shifts from arithmetic to algebraic discourse in pre-adolescents
  - Nachlieli & Tabach (2012), Sinclair & Moss: difficulties in moving students from the object level to the meta-level.

Combining Commognition with other frameworks

- Sfard's papers overtly reject "cognitivism" (Sfard 2000a, 2007; Sfard & Lavie, 2005) and avoid using terms that have not been defined within the theoretical framework.

- Other authors are less explicit about critiquing "cognitivist" ideas (Jankvist, 2011; Ryve, 2004; Xu & Clarke, 2013)

- Still there are issues with the "commensurability" of the commognitive framework with other frameworks and theories in mathematics education.

Interactional Socio-Linguistics

Interactional Socio-linguistics sub-heritage

- N = 18
- Draw on
  - Conversation Analysis, linguistic anthropology and writings of Ervin Goffman
  - Significant writers drawn on: John Gumprez, Frederick Erikson, Harvey Sacks, Hugh Mehan
- Early birds: Lampert (1990), O'Connor & Michaels (1993)

Interactional socio-linguistics - Activities

- Often had a dual goal:
  - Introducing a methodological tool or theoretical concept
  - Exemplifying a productive teaching practice, often aligned with NCTM documents (ex. Lampert, 1990)
- Some focused only on
  - A particular method of linguistic analysis
  - Applying ideas from sociolinguistics to problems of instruction
- Object of inquiry
  - Classroom talk, mostly in public schools, mostly k-12
  - Some looked at very specific levels such as intonation and prosody (Forrester & Pike, 1998; Staats, 2008)
  - Others looked at much broader aspects of activity (ex. time allocated for tasks (Jurow, 2005; Lampert, 1990)

Main concepts and ideas

- Revoicing (O'Connor and Michaels, 1993)
- Participant frameworks (Foman & Ansell, 2002)
- Animation (Empson, 2003)
- Focusing interactions (Lobato, 2012)

Examples of findings

- Offer sociolinguistic and ethnographic lenses to look at mathematics classrooms
  - Brilliant-Mills (1994): looks at what "counts as mathematics" in one particular classroom
  - Forrester and Pike (1998): using conversation analysis to uncover implicit ideas surrounding the teaching and learning of measurement and estimation in a 6th grade classroom.

- Sometimes used in combination with other frameworks
  - Lobato (2012): used Goodwin's (1994) "focusing interactions" and Grounded Theory to explain transfer in the learning of linear slopes
  - Jurow (2005) combined "figured worlds" with "participation frameworks" and "footing" – examined engagement in simulated real-world project

Building tasks

- Foregrounds:
  - The activity of talk and the social frameworks that are constructed by talk.
  - Inscriptions in the classroom
  - Relationships between teacher and students
- Backgrounds:
  - Politics
 - Identities

Discussion

Mostly, this literature

- Focused on grade 2-10 classrooms
- Examined spoken language or interactions, rather than the reading or writing of texts or existing written texts or non-verbal aspects of classrooms;
- Engaged in detailed analyses of a rather small set of data, with few mixed-method or solely quantitative analyses; and
- Focused on *relationships* between teacher-students or students-students, rather than relationships between people and mathematics or adult learners

Some of this literature

- Has had an increased number of references in the literature

Commognition
Positioning
SFL
Sociocultural
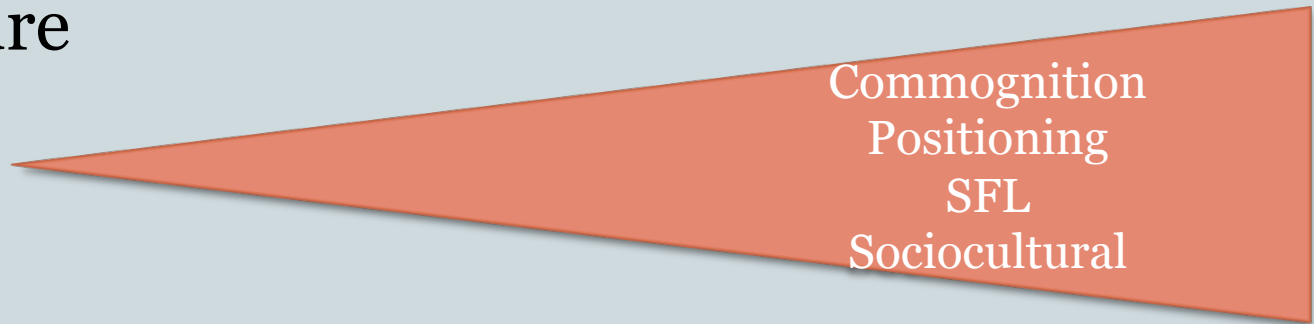
- while other perspectives have had a diminishing presence

Interactional sociolinguistics, Gricean, Threads

Geography of the literature

- Some of the sub-heritages appear almost exclusively in US-based journals (ex. Emergent/Symbolic interactionism)

- Others appear almost exclusively in journals outside of the US (Interactional sociolinguistics, Reconceptualists)
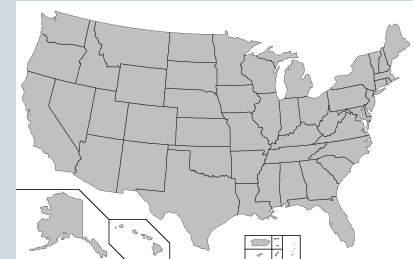
Methodological Issues

- Huge variation in the articulation and use of theory

- Only some articles operationalized their focal ideas

Both of these seemed to be getting better over time

- When authors put various ideas and theories together, little examination of commensurability

- Positionality generally absent in many articles
  - Sometimes described relevant 'roles,' but hardly ever addressed social identities

- Theories of interaction vs. theories of learning
 - Not enough work on the lamination between them

Reflection on Heritage Analysis

- ## Sign systems: what is discourse?

  - Discourse and related terms only sometimes defined

  - Discourse sometimes a *lens with which to understand some other phenomenon* (e.g., mathematical understandings, beliefs)

  - and is sometimes treated *as the object of inquiry*.

  - Many sub-heritages have created 'math' and 'non-math' discourse categories.
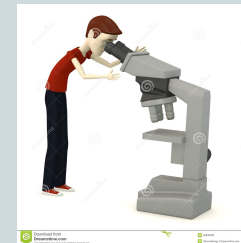
Scientific/ NonScientific concepts

Math/ Everyday discourse

? Math register/ other?

Reflection on Heritage Analysis

- *Signs and systems of knowledge*: What is mathematics?
  - Answers vary tremendously
    - Often absent or implicit
  - Discursive psychology: explicit relativist stance
  - Sociocultural, some Reconceptualist, Positioning, and SFL work: math as a social practice ('content' but also ways of being, interpersonal, power dynamics, history)
  - Emergent and some Sociocultural: seemed to rely on quasi-empiricist philosophy
  - Reconceptualists: most critical of narrow views of status-quo school math

Reflection on Heritage Analysis

- *Identity* is mainly attended to by Positioning and Reconceptualist sub-heritages (a little in Commognition)

- *Power* also mainly attended to by Positioning and Reconceptualist subheritages (some in SFL, too)

Reflection on Heritage Analysis

- *Activities:*
  - Some sub-heritages still spending a lot of time justifying their ideas/approaches; others do not
    - Indicator of what is "common knowledge" or status quo in the field?
  - Reliance on policy documents like NCTM
  - Emphasis on "inquiry-based" mathematics
    - Few explicitly questioning 'school mathematics'
    - Few tending to inconsistencies between constructivism and discursive theories
  - Curious absence of arguments (and data) related to achievement outcomes and measurement

Thank you!