Building a Knowledge Base and Intellectual Capacity in Mathematics Education:

Promises and Challenges

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Outline

- Three Reflections
- New NCTM Research Handbook
- LieCal Project
- Mathematical Model of Marriage

Reflection 1

Math Education Researcher

First, Cross-national Comparative Studies

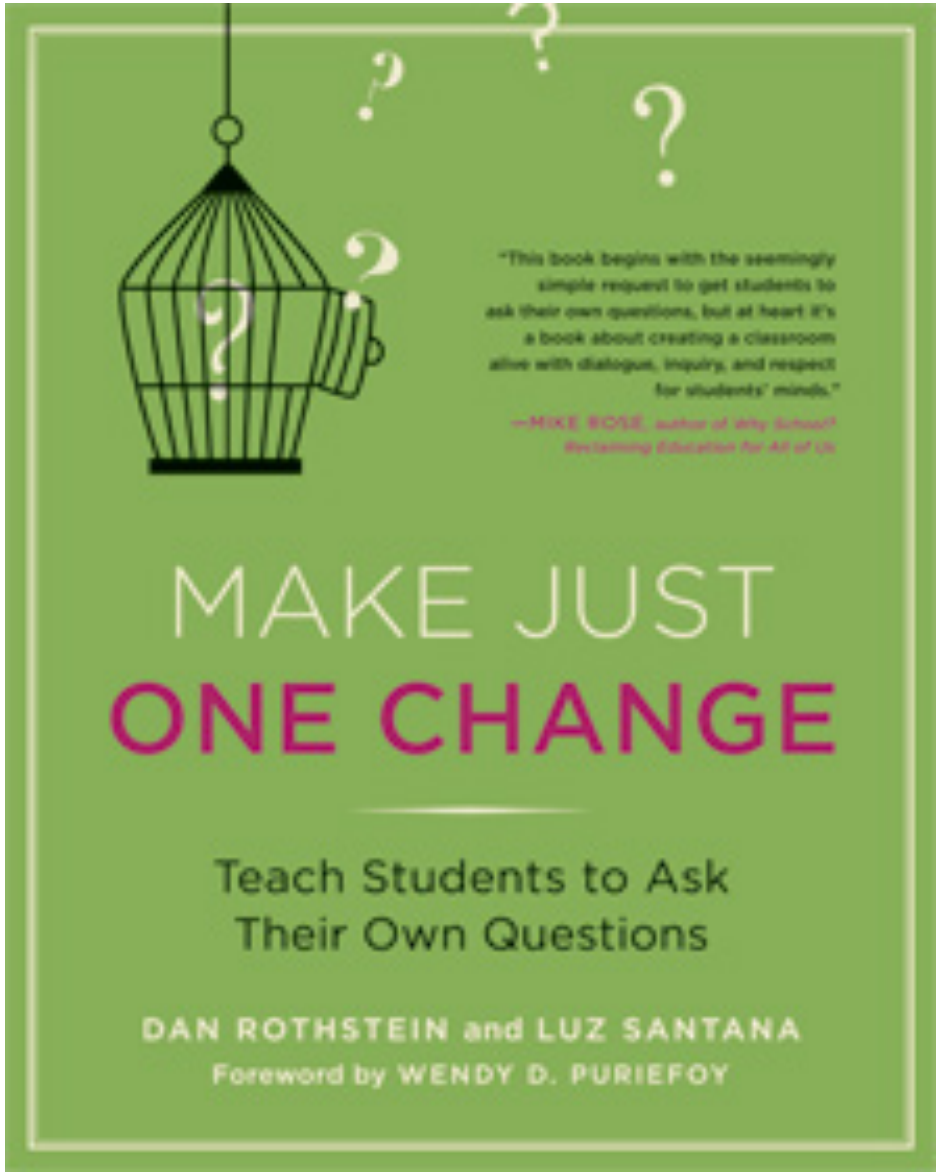
Second, Curriculum Studies

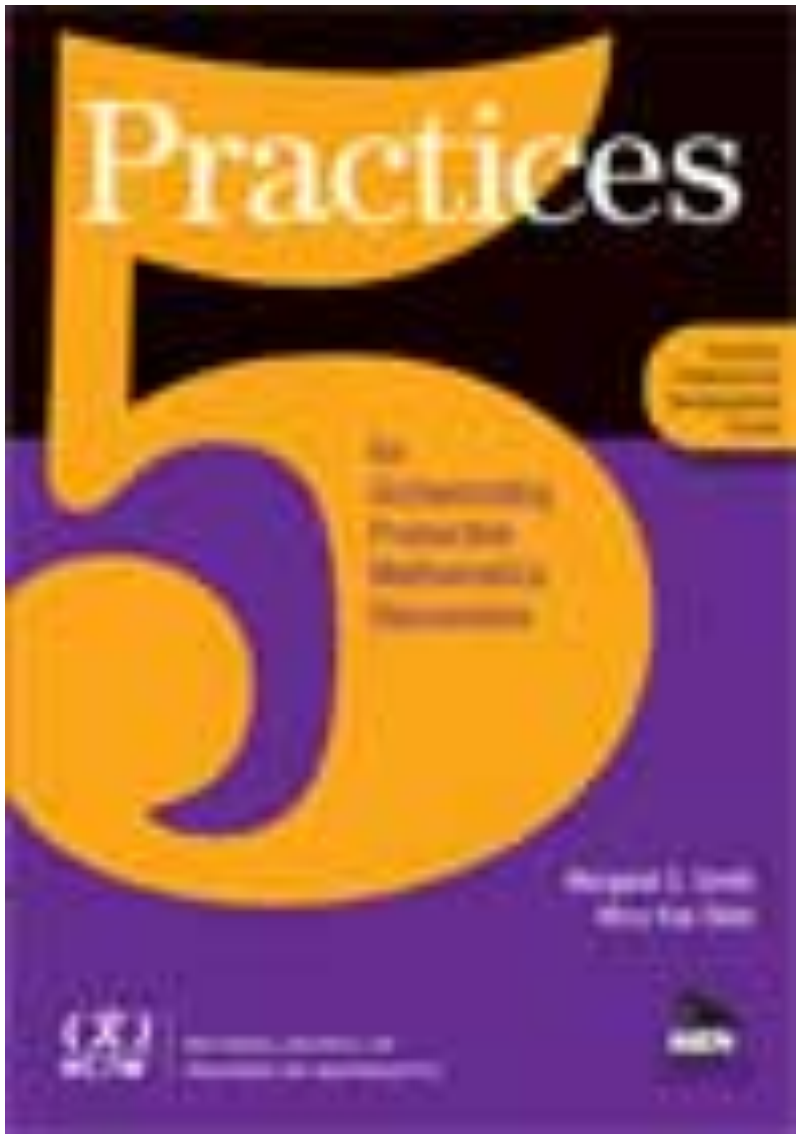
Third, Mathematical Exploration

Reflection 2

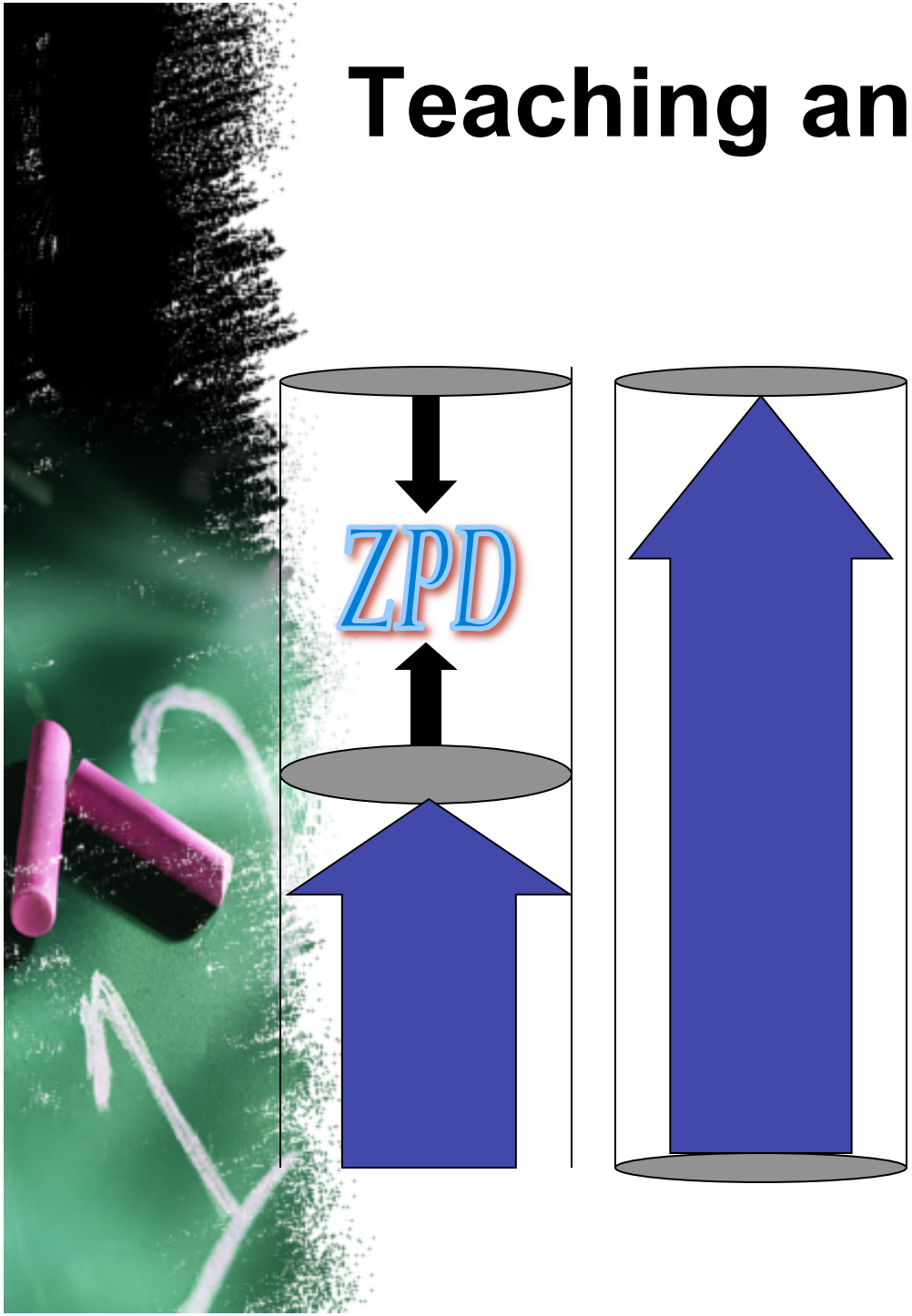
Teacher Educator

- Many Reform Ideas
- Many Theories


"This book begins with the seemingly simple request to get students to ask their own questions, but at heart it's a book about creating a classroom alive with dialogue, inquiry, and respect for students' minds."
—MIKE ROSE, author of Why School? Reclaiming Education for All of Us
MAKE JUST
ONE CHANGE
Teach Students to Ask Their Own Questions
DAN ROTHSTEIN and LUZ SANTANA
Foreword by WENDY D. PURIEFOY


Practices

Teaching and Learning



Educational Setting

ZPD

Natural Setting

Reflection 3

NSF Program Director

- Elevator talk
- IES and NSF Common Guidelines for Education Research and Development

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Research Handbooks

- **Cited Very Frequently**
- **Similar Structure**
- **Similar Topics**

Research Handbooks

- New Topics
- New Sections
- New Consideration of Author Teams

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- Three Reflections
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Longitudinal Investigation of the Effect of Curriculum on Algebra Learning (LieCal Project)

Project Team

PIs

- Jinfa Cai John C. Moyer Ning Wang

Project Coordinators at the Research Site

- Pat Bolter/Victoria Robison

Research Associates/Specialists/Graduate Assistants

- Bikai Nie Tammy Garber
- Tony Freedman Stephen Hwang
- Yuichi Handa Connie Laughlin
- Patrick Hopfensperger Yue Zeng Steve Silber
- Teresa Lupia Jia Mi Matt Wells

Undergraduate Assistants

- Maria Alyson Carole Bryne
- Sonya Poirier Chelsey Schwander
- Kim Rubin Carly Toth

LieCal Project History

6th	7th	8th	9th	10th	11th	12th
←--------	2005-	2009	-------→		←2010-	2012→

Purposes

- A profile of the intended treatment of algebra in the CMP curriculum with a contrasting profile of the intended treatment of algebra in the non-CMP curricula;
- A profile of classroom experiences that CMP students and teachers have, with a contrasting profile of experiences in non-CMP classrooms; and
- A profile of student performance resulting from the use of the CMP curriculum, with a contrasting profile of student performance resulting from the use of non-CMP curricula.

Research Site

- A Larger Urban School District
- 51 schools in the district have students in the middle grades: 27 use CMP and 24 use non-CMP

Research Site (cont.)

- Diverse student population:

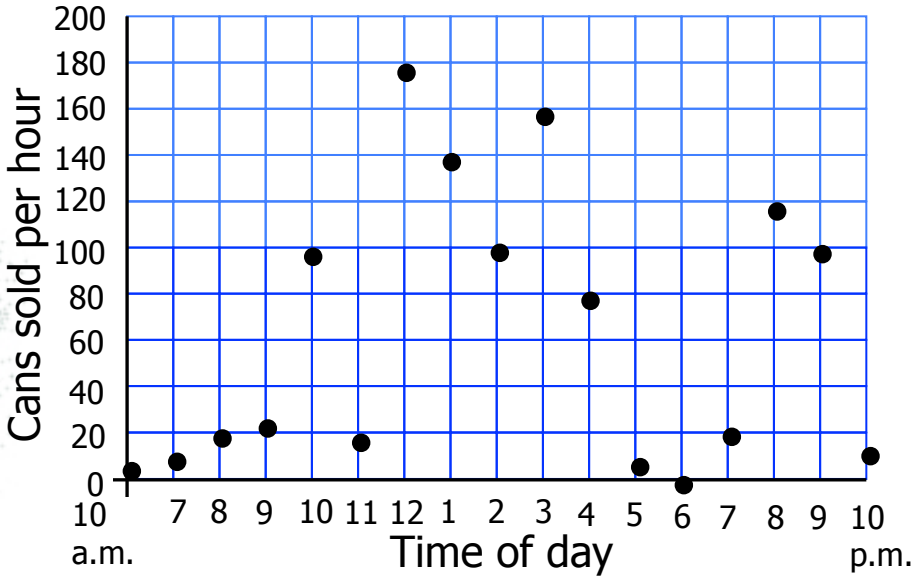
  - 62% African Americans
  - 21% Hispanic,
  - 12% white,
  - 4% Asian, and
  - 1% Native Americans

Profile of Schools

Achievement Level	CMP	Non-CMP
High Achieving	2	2
Average Achieving	3	3
Low Achieving	2	2

A sample problem in CMP curriculum

The graph below shows the numbers of cans of soft drink purchased each hour from school's vending machine in one day (6 means the time from 5:00 to 6:00, 7 represents the time from 6:00 to 7:00, and so on).



a. The graph shows the relationship between two variables. **What are the variables**?

b. Describe how the number of cans sold **changed** during the day. Give an explanation for why these changes might have occurred.

Sample problems in a US Non-CMP curriculum

Evaluate algebraic expressions:

(1)Evaluate $16+b$ if $b=25$.

(2)Evaluate $x-y$ if $x=64$ and $y=27$

Identify the solution of an equation :

$9+w = 17$; choose one from 7, 8, 9

How is variable defined?

- "A variable is a quantity that changes or varies."

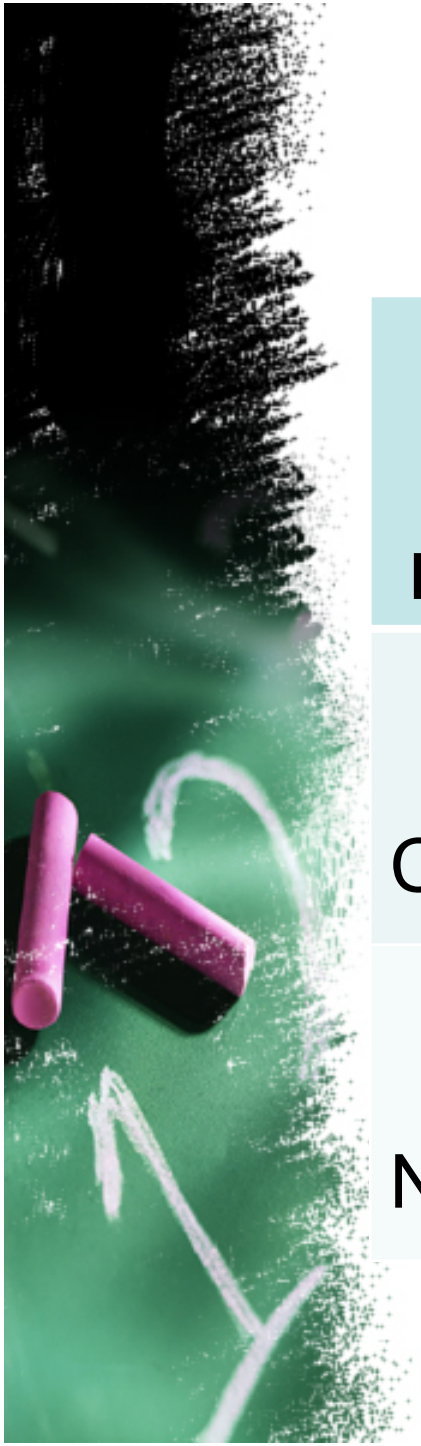
  (*CMP*)

- "A variable is a symbol, usually a letter, used to represent a number. "

  (Non-CMP)

How is equation defined?

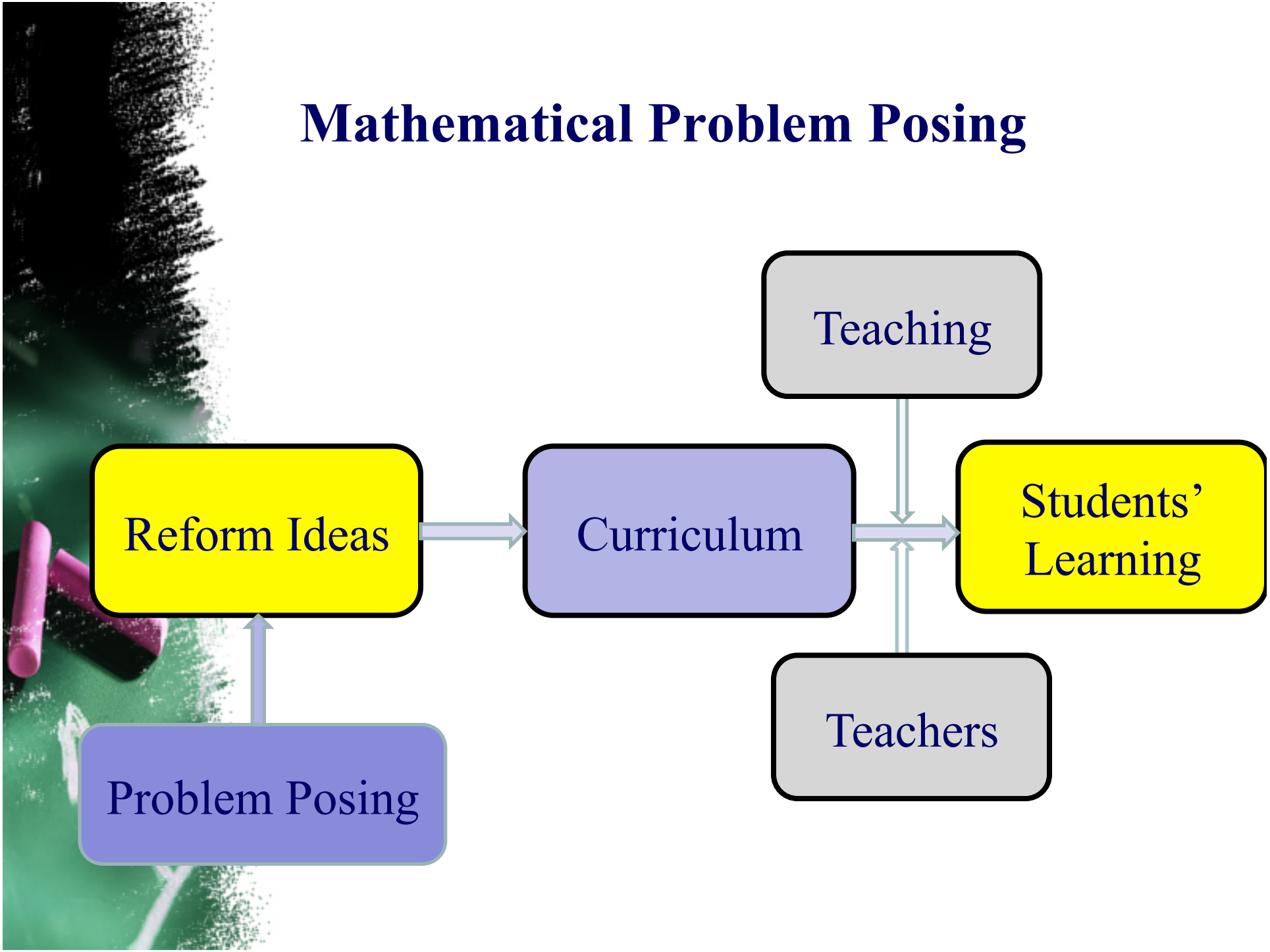
- Rather than seeing equations simply as objects to manipulate, students are shown that equations often describe relationships between varying quantities that arise from meaningful, contextualized situations. (*CMP*)
- "…a sentence that contains an equals sign, =" (Non-CMP)



% of Problems Involving Linear Equations

Types of Problems	1equ 1va (x+2=5x)	1equ 2va (y=3x +4)	2equ 2va (3y=x+2) & (y=5x+9)
CMP	5.72	93.03	1.24
Non-CMP	86.19	11.67	2.14

Mathematical Problem Posing



Number of PP tasks in Different Grade Levels

Grade	Chinese – BNU		US -- Investigations	
	Total tasks	% PP	Total tasks	% PP
1	570	5.96	490	0
2	549	5.65	741	1.62
3	541	2.77	832	0.96
4	561	2.85	760	2.24
5	619	2.91	726	3.17
6	545	3.12	-- [1]	--
Total	3,385	3.87	3,549	1.69

[1] The Investigations series does not have Grade-6 textbooks.

Distribution of PP tasks in different content areas

Content Area	Chinese (n=131)	US (n=60)
Numbers and Operations	61.07	78.33
Algebra	3.05	18.33
Geometry	2.29	0
Measurement	2.29	0
Data analysis and probability	12.98	3.33
Undetermined	18.32	0

Observations

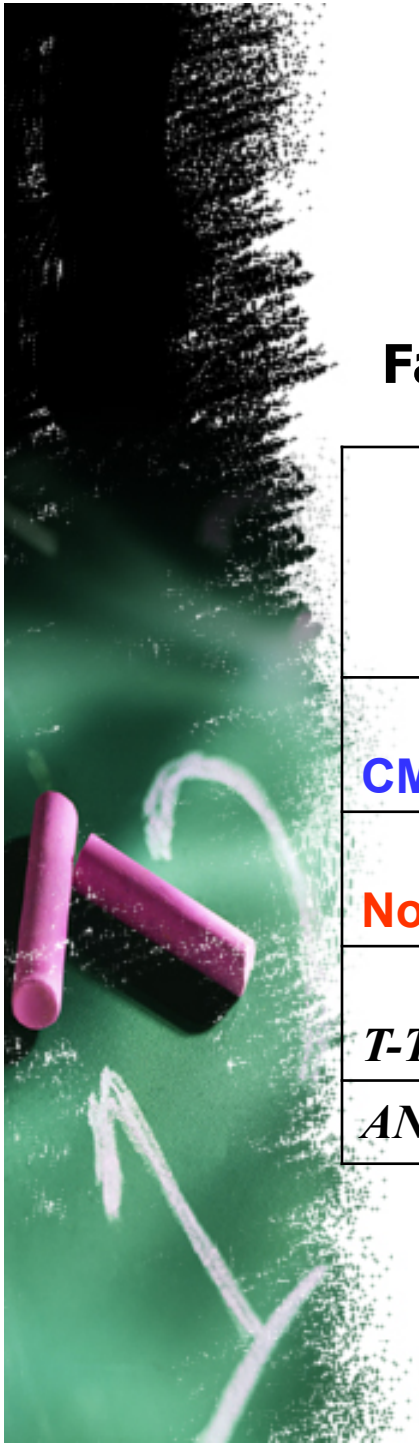
Background Information

- 50 sixth-grade classrooms
- 4 observations per classroom (2F, 2Sp)
- 2 trained observers (experienced math teachers) did the observations
- 3 reliability checks done during the year

Observation Instrument

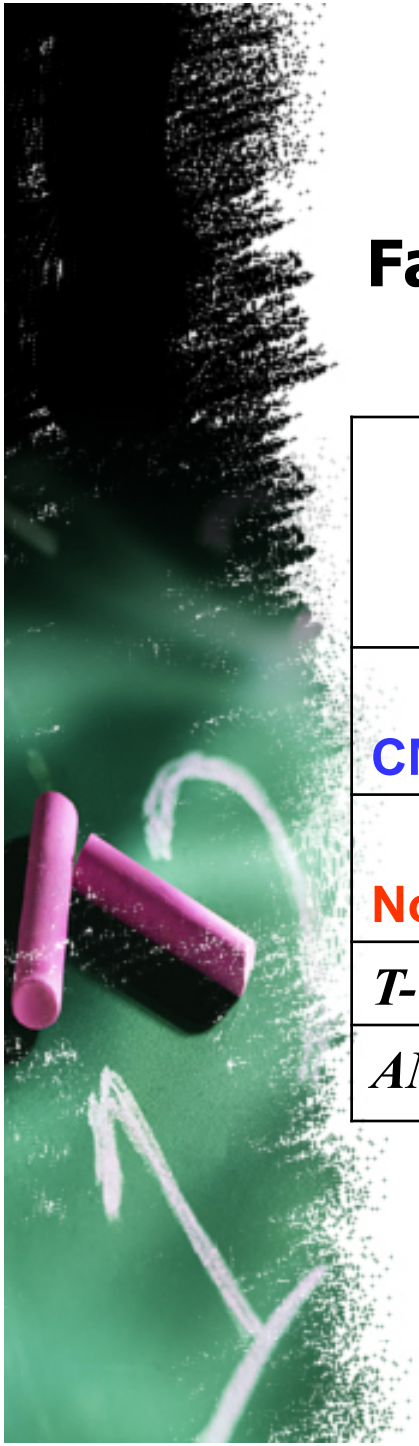
Main Components

- **Conceptual Emphases;**
- Procedural Emphases;
- **Instructional Tasks;**
- Homework Problems



Factor 1: Emphasis on Conceptual Understanding

	Grade 6 CMP:n=100; Non-CMP: n=95	**Grade 7** CMP:n=105; Non-CMP: n=103	**Grade 8** CMP:n=112; Non-CMP: n=100	**Total** CMP: n=317 Non-CMP: n=298
CMP	**17.99** (4.56)	**15.68** (4.34)	**16.88** (4.65)	**16.83** (4.60)
Non-CMP	**12.33** (3.13)	**13.60** (3.04)	**14.12** (3.71)	**13.37** (3.38)
T-Test	*P<.0001.*	*P<.0001.*	*P<.0001.*	*p<.0001.*
ANOVA:	*F (3, 611)=39.09, p<.0001.*			



Factor 2: Emphasis on Procedural Knowledge

	Grade 6 CMP: n=100; Non-CMP: n=95	**Grade 7** CMP: n=105; Non-CMP: n=103	**Grade 8** CMP: n=112; Non-CMP: n=100	**Total** CMP: n=317 Non-CMP: n=298
CMP	**14.70** (3.66)	**14.41** (3.72)	**15.25** (4.18)	**14.80** (3.88)
Non-CMP	**17.16** (4.41)	**17.72** (4.12)	**18.33** (3.97)	**17.75** (4.18)
T-Test	*P<.0001.*	*P<.0001.*	*P<.0001.*	*p<.0001.*
ANOVA:	*F(3, 611)=29.38, p<.0001.*			

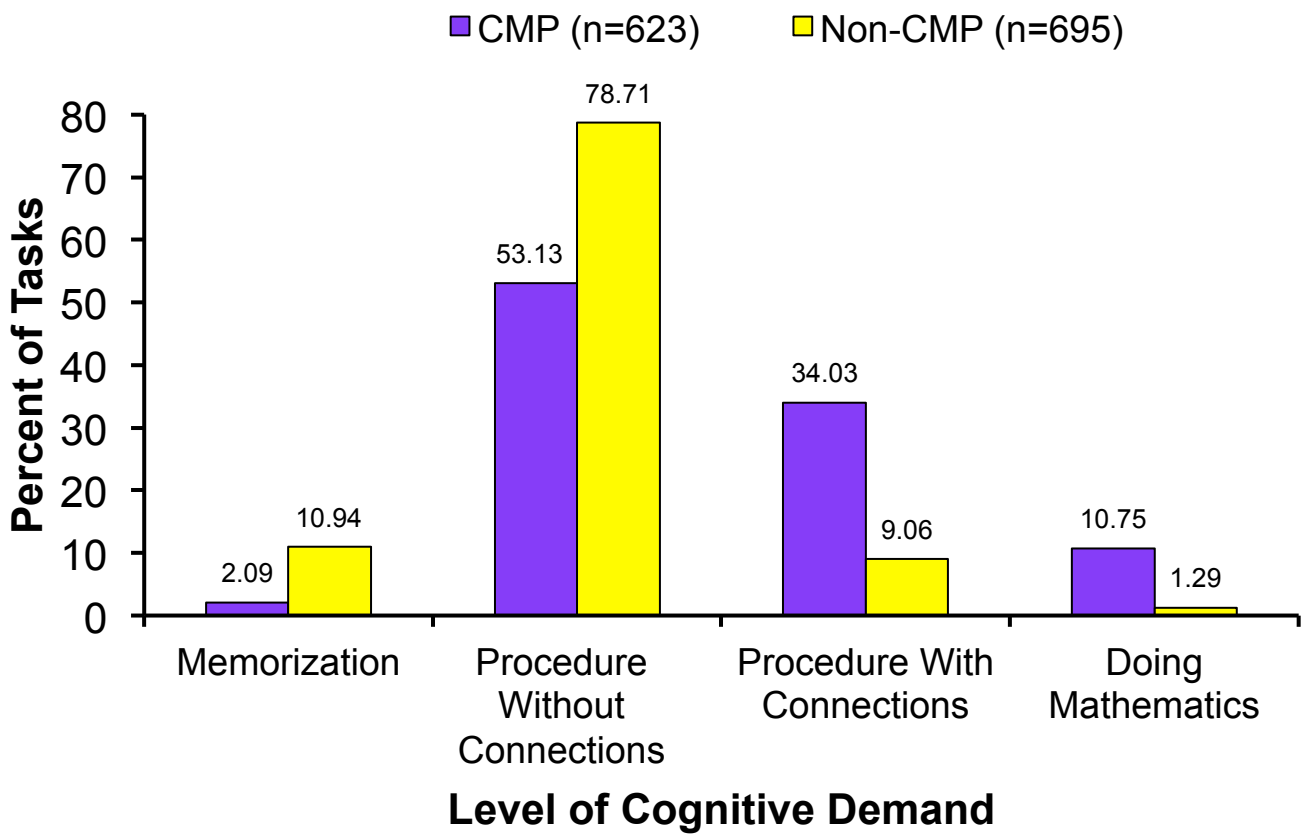
The cognitive level of the instructional tasks implemented

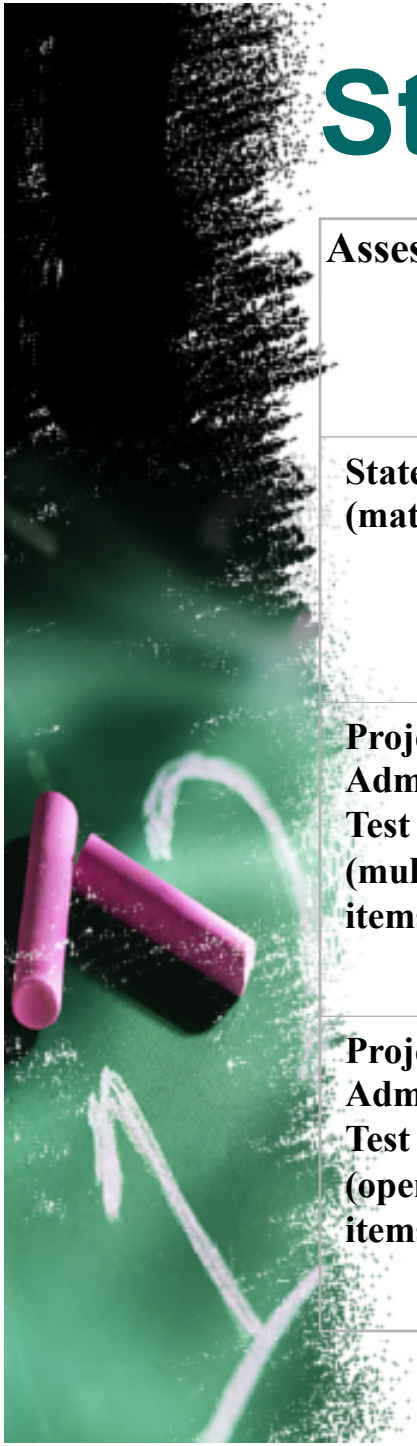
Instructional tasks were categorized into four increasingly demanding levels of cognition(Stein et al.,1996):

- Memorization;
- Procedures without connections;
- Procedures with connections; and
- Doing mathematics.

The Cognitive Level of the Instructional tasks Implemented

The Percentage Distributions of the Cognitive Demand of the Instructional Tasks

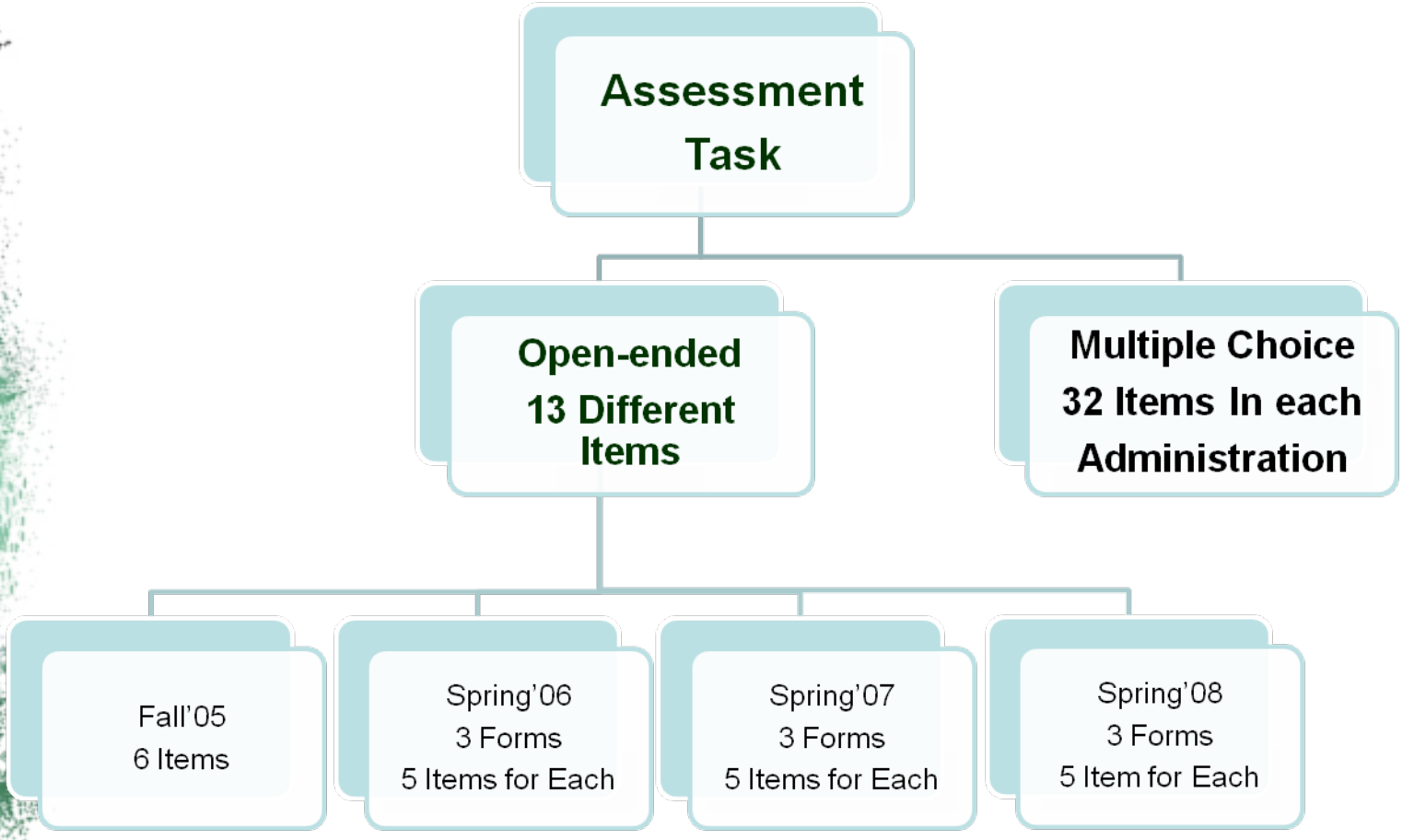




Student Assessment: Time Table

Assessments	Fall (05-06)	Spring (05-06)	Fall (06-07)	Spring (06-07)	Fall (07-08)	Spring (07-08)
State Tests (math & reading)	All students		All students		All students	
Project-Administered Test (multiple-choice items)	6th grade students	6th grade students		7th grade students		8th grade students
Project-Administered Test (open-ended items)	6th grade students	6th grade students		7th grade students		8th grade students

Project-Administered Student Assessment Components



Achievement Scaled Scores

- A scaled score is a generic term for a mathematically transformed student raw score on an assessment.
- Using scaled scores, rather than raw scores, assessment results can be placed on the same scale even though students responded to different tasks and at different times.
- The two-parameter Item Response Theory (IRT) model was used to scale student assessment data.

Achievement Scaled Scores

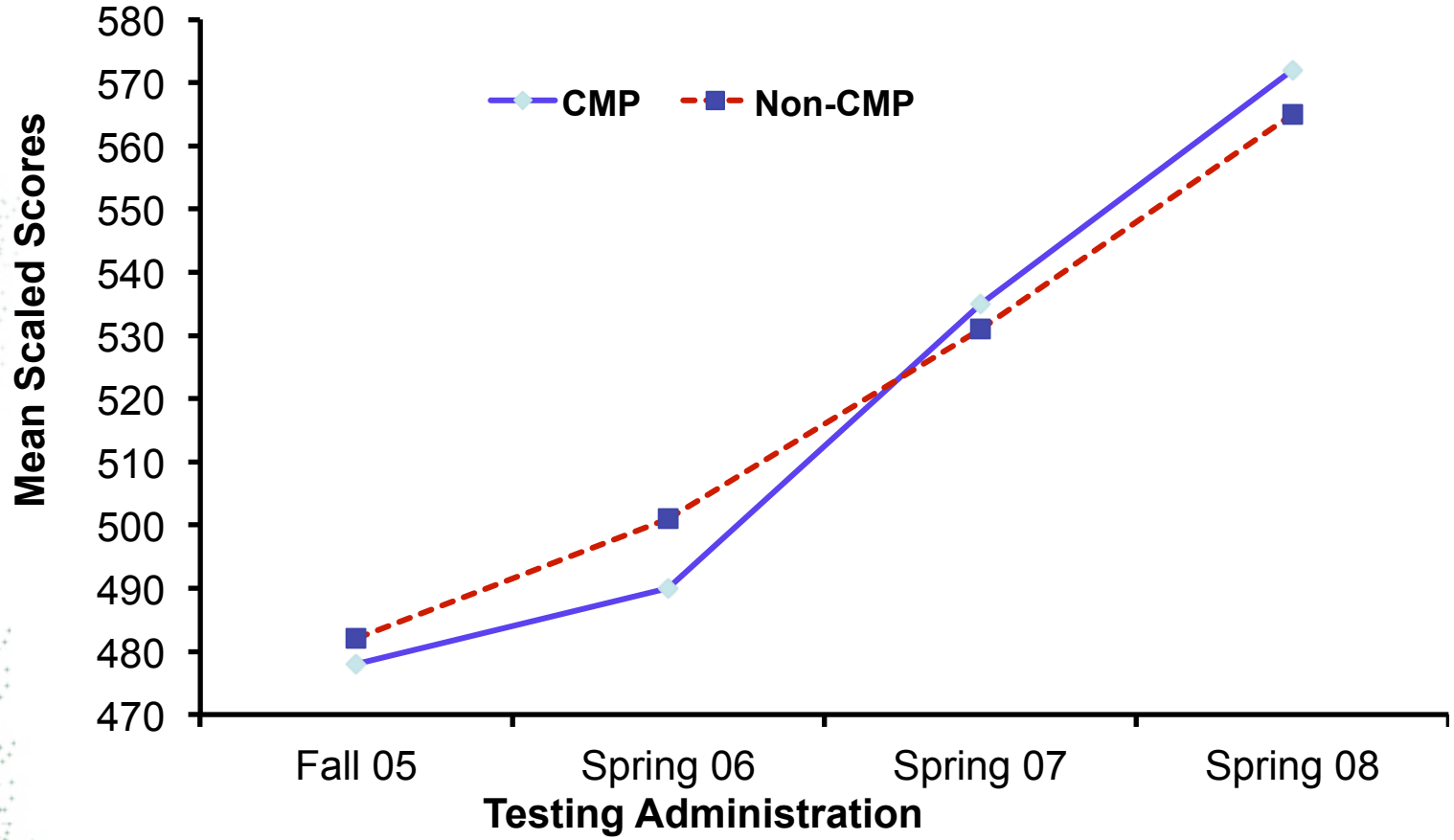
- The two parameters are: An item difficulty index (easy or hard item) and an item discrimination index (how well an item distinguishes lower from higher achievers).
- Using the two-parameter IRT model, student responses were scaled across all forms and three assessment times.

Quantitative Analysis of student achievement data

- *Repeated Measures ANOVA*
- *ANCOVA*
- *HLM Growth Curve Modeling*
- *HLM Cross-Sectional*

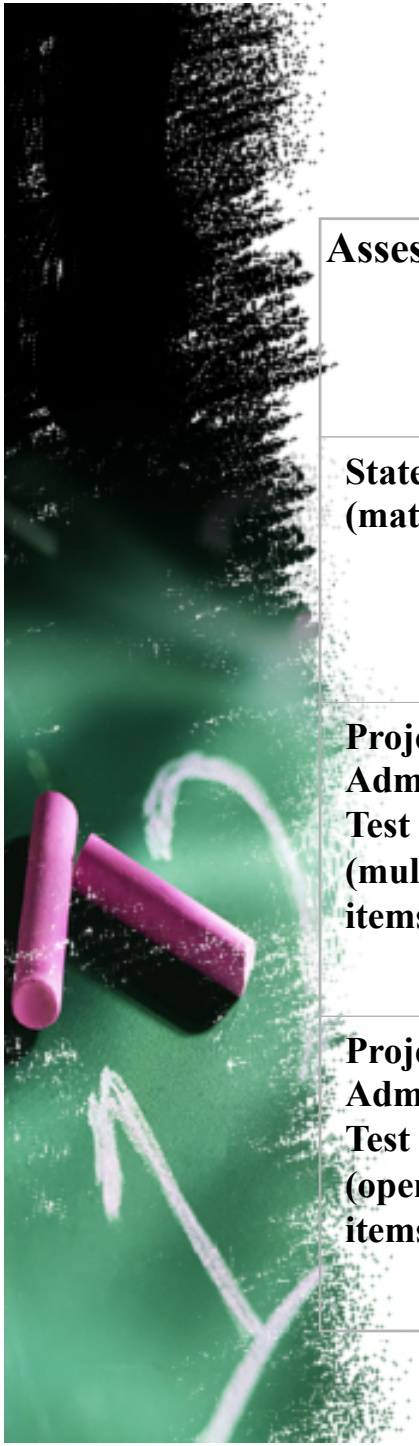
A Case from LieCal Project

Mean Scores for CMP and non-CMP Students on the Open-ended Tasks



LieCal Project History

6th	7th	8th	9th	10th	11th	12th
←--------	2005-	2009	-------→		←2010-	2012→



Data Source

Assessments	Fall 6th grade	Spring 6th grade	Fall 7th grade	Spring 7th grade	Fall 8th grade	Spring 8th grade
State Tests (math & reading)	All students		All students		All students	
Project-Administered Test (multiple-choice items)	6th grade students	6th grade students		7th grade students		8th grade students
Project-Administered Test (open-ended items)	6th grade students	6th grade students		7th grade students		8th grade students

Data Source

Data Source	9th Grade	10th Grade	11th Grade	12th Grade
Open-ended tasks			√	√
Multiple choice tasks			√	√
State Test Data		√		
Mathematics Grades	√	√	√	√
Enrollment in Advanced Math Courses			√	√
SAT/ACT Registration and Scores			√	√

10th Grade State Test Covariate(s)	F-Value
PI-developed 6th grade MC tasks	5.13*
PI-developed 6th grade OE tasks	3.90*
6th grade State math test scaled score	9.58**
7th grade State math test scaled score	9.57**
8th grade State math test scaled score	11.79***

Problem Posing and PS Strategies

- Posing similar or more complex problems
- More Abstract strategies

Research in Medical Education

- PBL v.s. Lecture
- Immediate assessment

Knowledge: Lecture > PBL

Clinical: PBL > Lecture

- Delayed assessment

Knowledge: PBL > Lecture

Clinical: PBL > Lecture

Some Research Findings

(Cai & Merlino, 2011)

- A total of 1316 high school students
- Different programs:

  285 Non-college preparation mathematics

  858 college preparation math (traditional)

  173 college preparation math (NSF-Funded)

Survey Instrument

We are interested in learning how you think and feel about mathematics. Please take a few minutes to think about the following questions and write how you truly feel. There are no right or wrong answers.

- If Math were a **food**, it would be_because_____

  _____

- If Math were a **color**, it would be_because_____

  _____

- If Math were an **animal**, it would be_because

To show they like mathematics

- *"Purple is my favorite color. It's my birth stone color plus it brings passionate. That's how I feel about math."*

To show they like mathematics (Why?)

- *"Math is like steak because math is a full, expansive subject. However, like a steak there are tough bits of gristle scattered throughout obstacles you must work around. The full meal is satisfying, but the process of eating is somewhat unusually strenuous."*
- *"Vegetables are good for you, and so is mathematics for daily things. It is needed in life. Some people like it, and some people don't, but you still need it to live a healthy life."*

To show they dislike mathematics

- *"I would say a mosquito, because whatever you do to try and get away from it, it always comes back. It's annoying because you hate taking math every year, and whatever you try to do to stop it, it always fails."*

To show they dislike math (Why?)

- *"It is like gum. You chew gum and use it to freshen up your breath, but in the end, it's worthless and doesn't have any nutrition or vitamins. Math is used in school to determine your intelligence, but there is no need for it later."*

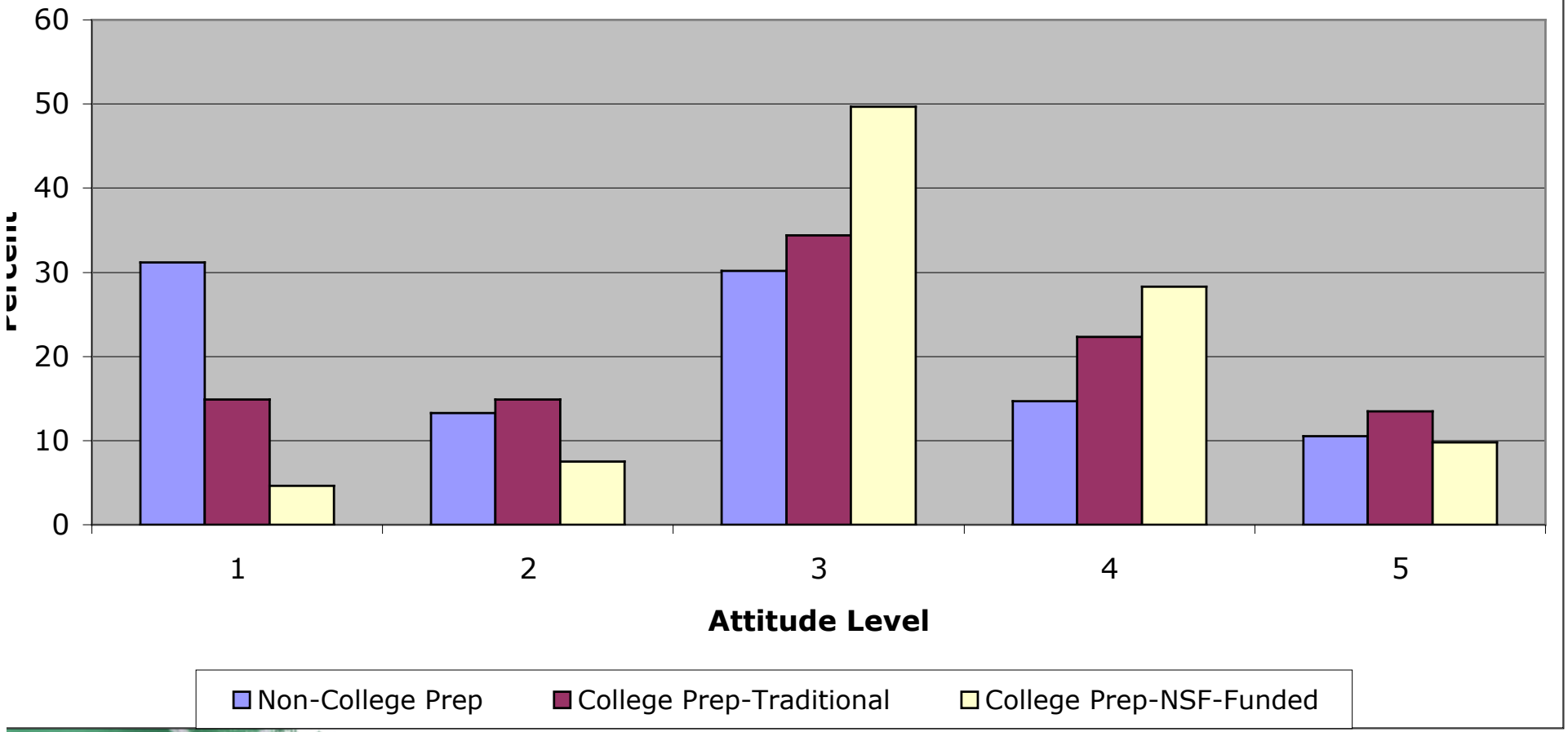
Analyses of Responses

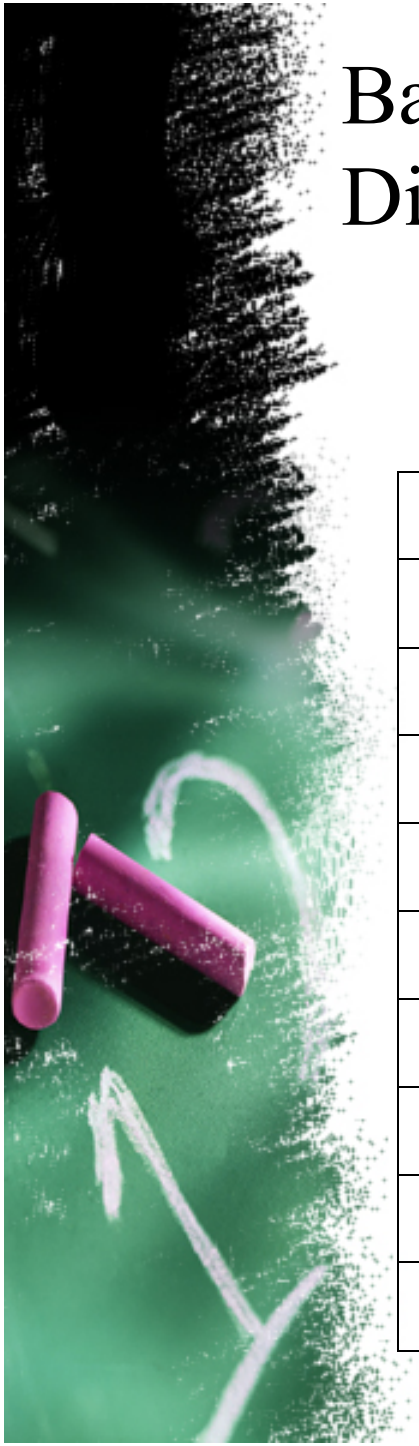
- Quantitative Analysis: Holistic scoring (1 - 5)

1 Point	Very Negative
2 points	Moderately Negative
3 points	Neutral or Ambivalent
4 points	Moderately Positive
5 points	Very Positive.

- Qualitative Analysis: Reveal what kinds of metaphors students used and why

Figure 2. Percentage Distribution at Each Attitude Level





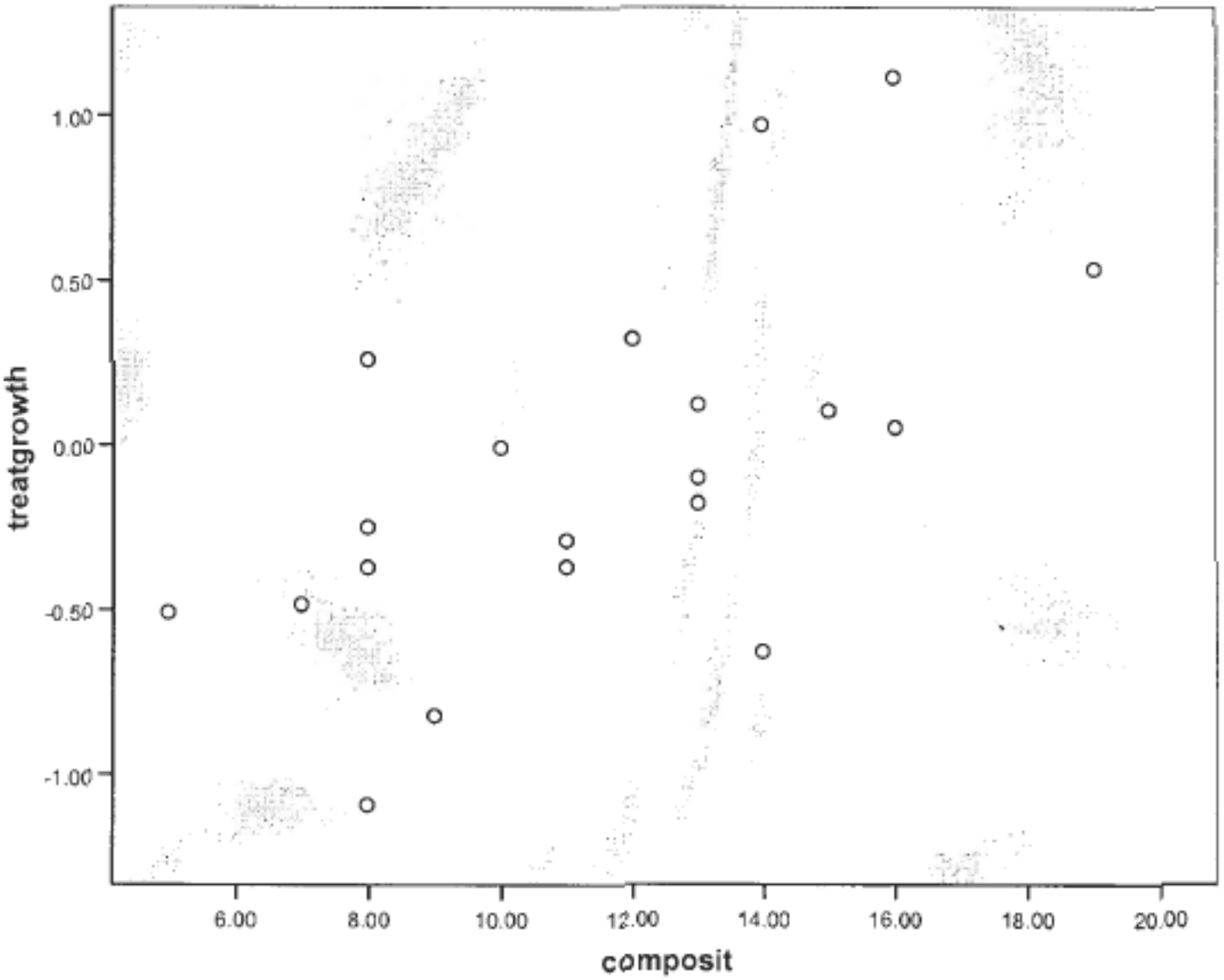
Background Information in the Ten School Districts in GPSMP (Kramer, Cai, & Merlino, in press)

	School District	Curriculum (20 Middle School)	Approximate # of Students (Middle School)	Curriculum (High School)	Approximate # of Students (12 High School)
1	District A (PA)	CMP	4000	CPMP	5000
2	District B (PA)	MiC	2000	CPMP	2000
3	District C (PA)	MiC	1000	IMP	2000
4	District D (PA)	MiC	1000	IMP	2000
5	District E (NJ)	CMP	500	CPMP	500
6	District F (PA)	MiC	1000	IMP	1000
7	District G (NJ)	CMP	1000	CPMP	1000
8	District H (NJ)	CMP	1000	IMP	2000
9	District I (PA)	CMP	1000	IMP	2000
10	District J (PA)	CMP	1000	CPMP	2000

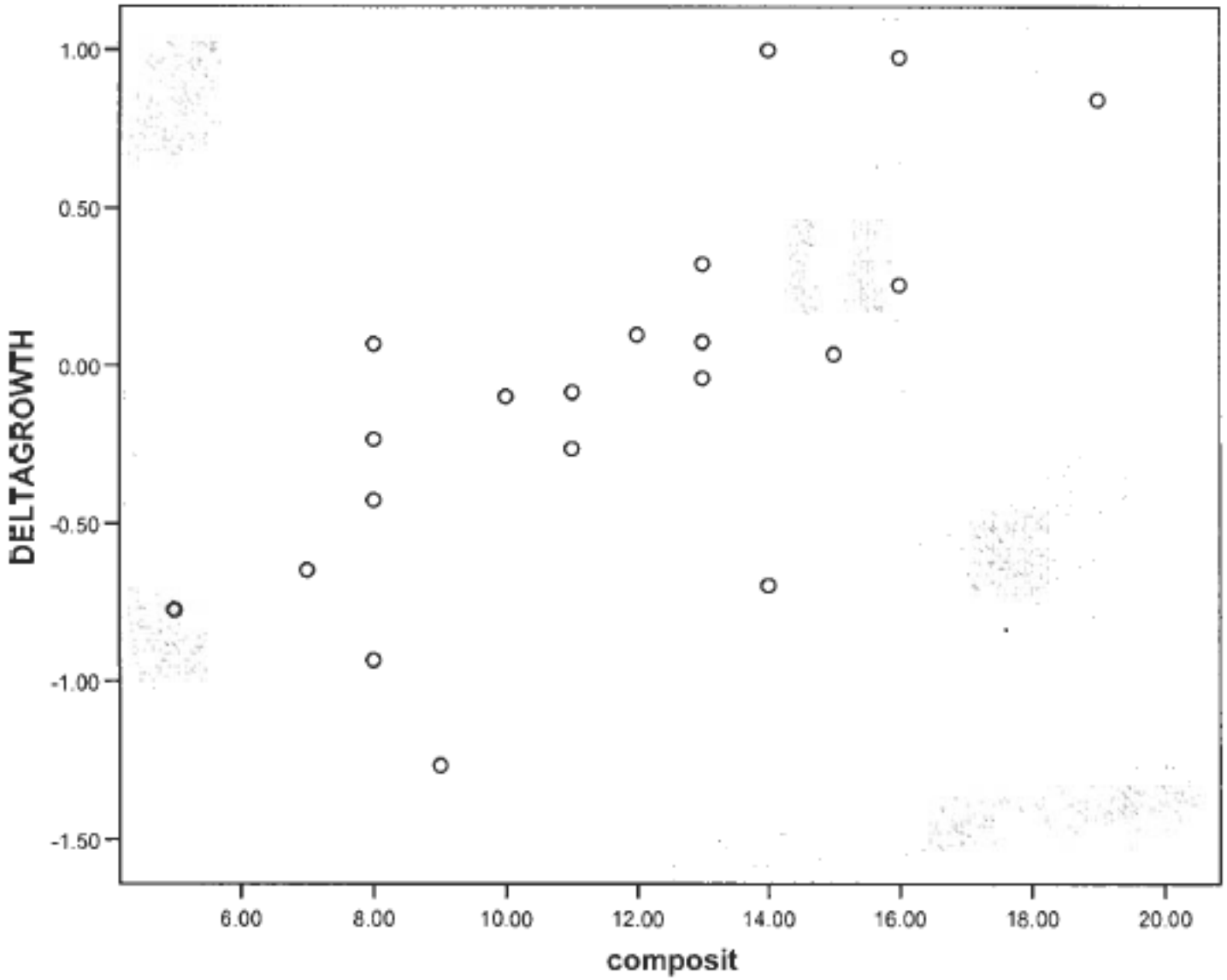
Four Factors for "Will to Reform"

- Superintendent support for the reform program;
- Principal support for the reform program;
- Teacher "buy-in" to the reform program;
- Coherence of School District support for the reform program

Scatter-plot of "Treatment Growth" (zmath04-zmath98 in PA; zmath04-zmath99 in NJ)



Scatter-plot of "Treatment Growth" - "Control Growth"



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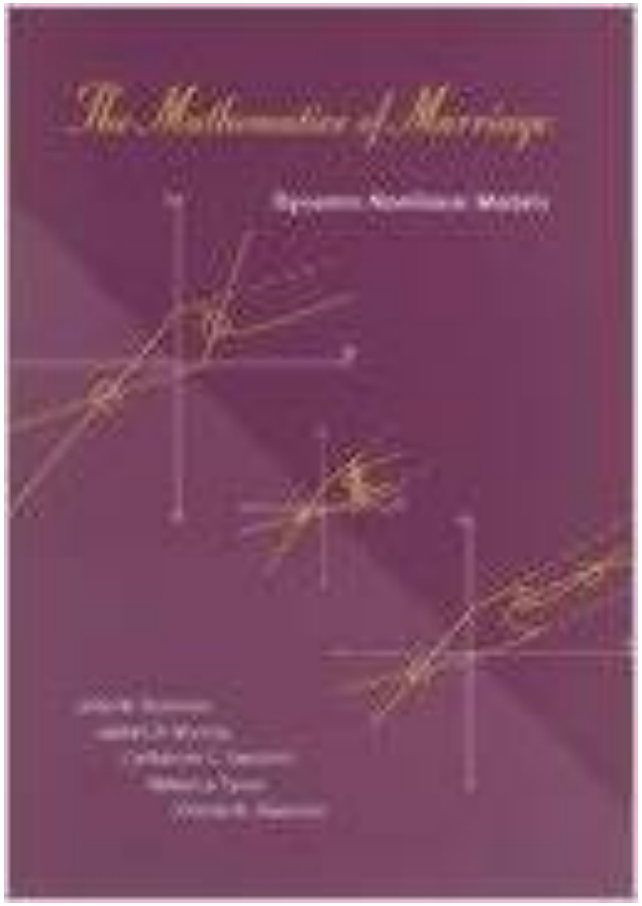
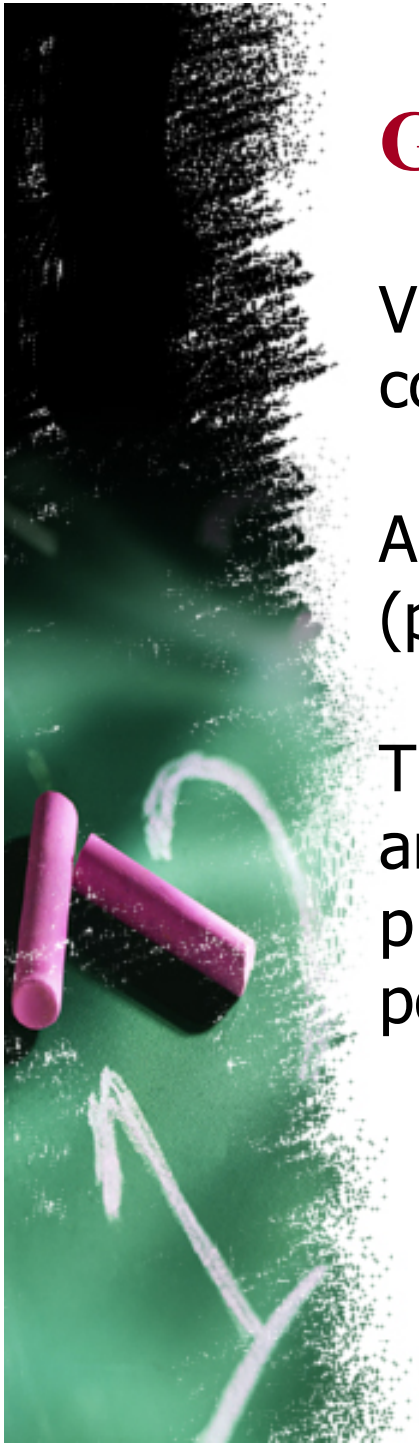
The Marriage Equation: A practical theory for predicting divorce & scientifically-based marital therapy

John Gottman and James D. Murray





Gathering a Couple's Data

Video is taken of the couple discussing a topic of contention, such as money, sex, housing, in-laws etc.

An accepted scoring system assigns a specific number (positive or negative) to each statement.

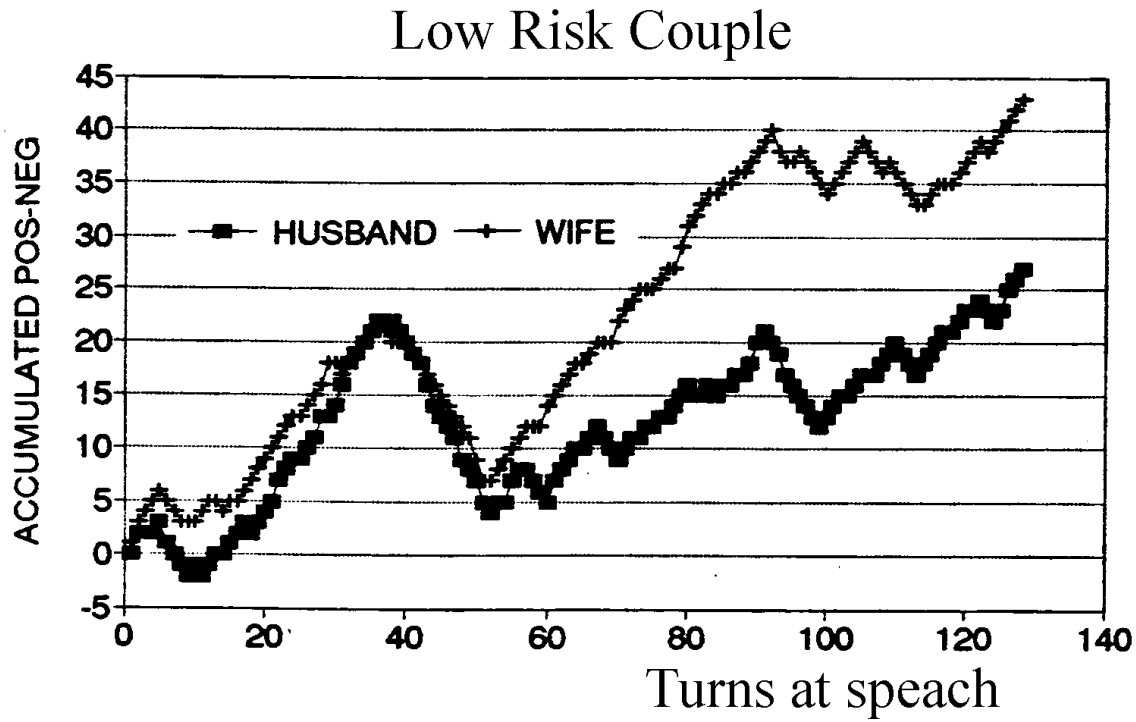
The scores (positive – negative) for the husband (H) and the wife (W) for each turn of speech (t) are plotted as functions of time. It measures the average positivity of each spouse as a function of time (t).

Data Representation: Typical Data for Low Risk Couple

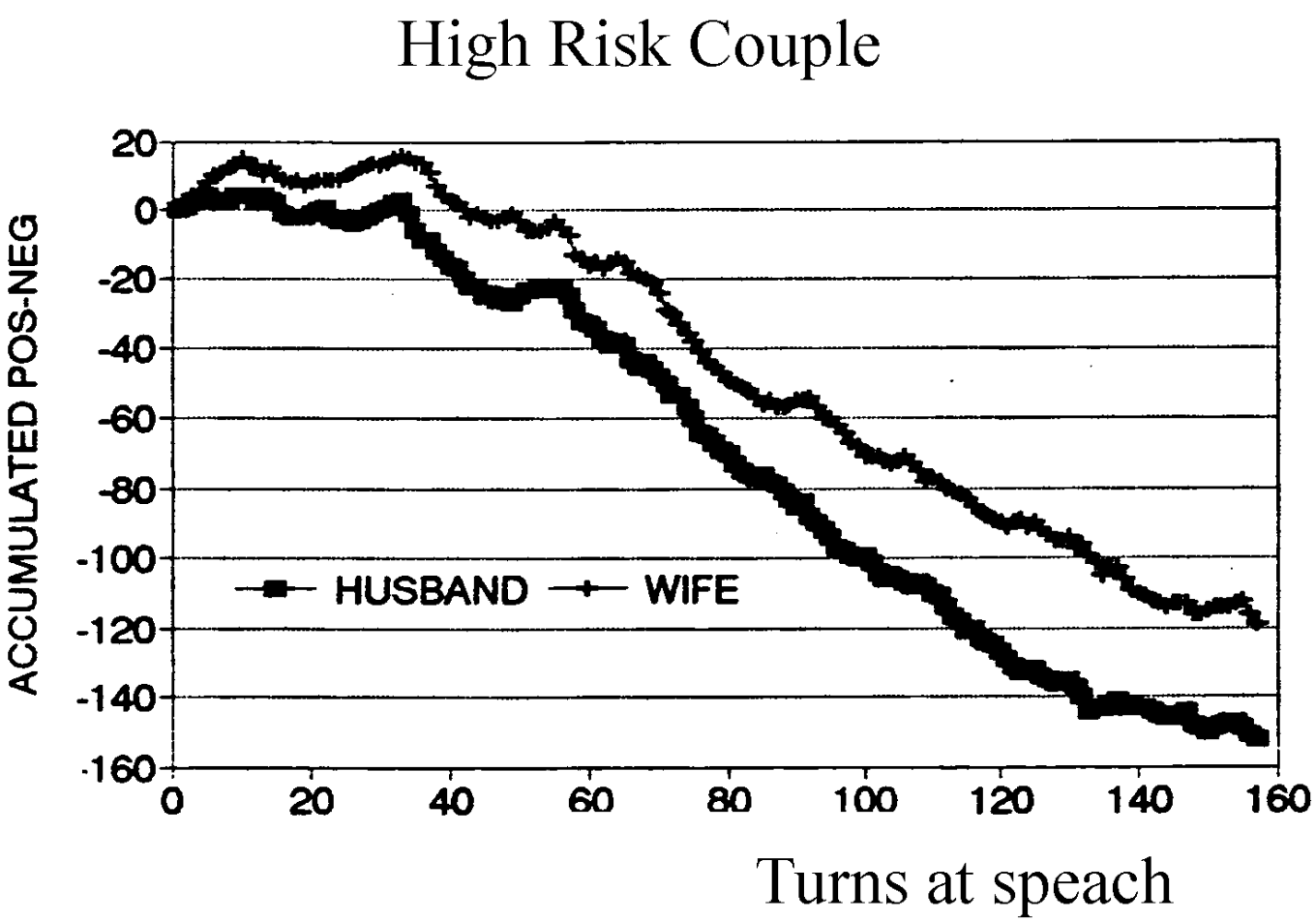
Cumulative "positive-negative" scores for each turn of speech for the husband and wife.

Stable marriage

Examples:
affection +4
disgust -3
whining -1
contempt -4



Typical High Risk Couple's Interaction



Unstable marriage

0.8 to 1 positive to negative ratio

Mathematical Model of the interaction

Wife's score at time t + 1	=	Constant	+	Wife's previous score	+	Husband's influence on Wife

$$W_{t+1} = a + r_1 W_t + I_{HW}(H_t)$$

Husband's score at time t + 1	=	Constant	+	Husband's previous score	+	Wife's influence on Husband

$$H_{t+1} = b + r_2 H_t + I_{WH}(W_t)$$

Basic Marriage Types

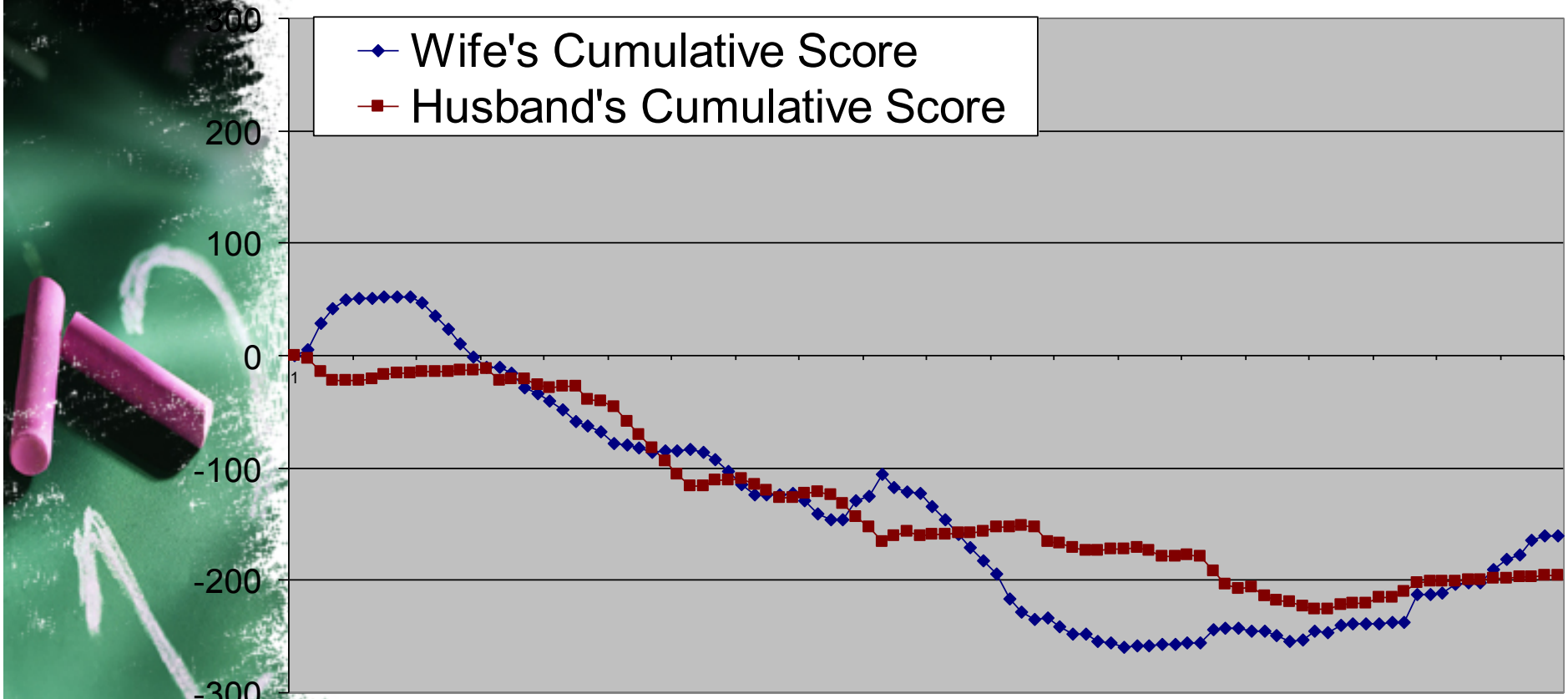
Observations of couples (**RCISS** - Rapid Couples Interacting Scoring System) and mathematical model **5 types of marriages:**

3 stable: (1) Volatiles, (2) Validators, (3) Avoiders

2 unstable: (1) Hostiles, (2) Hostile-Detached

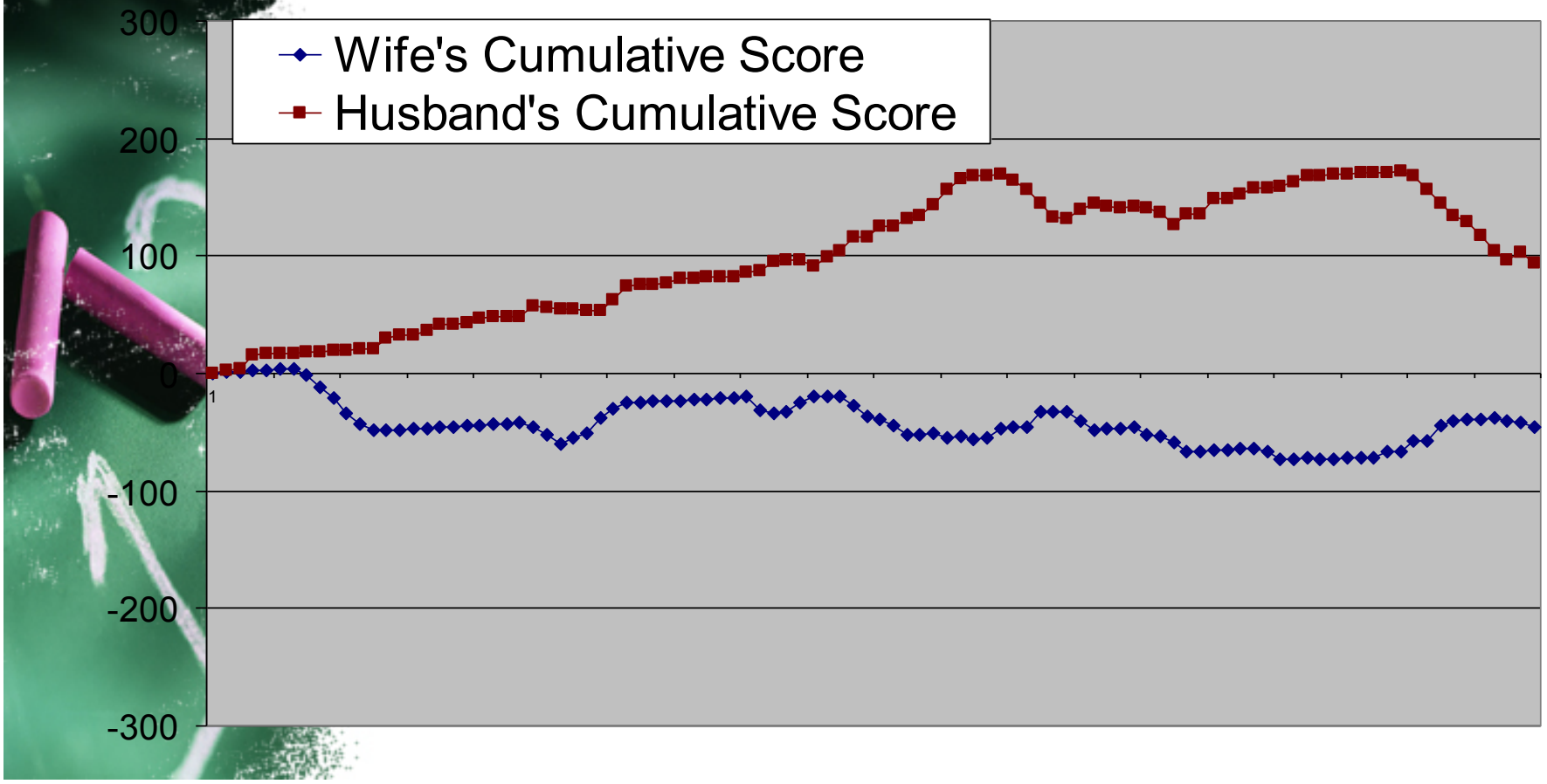
Pre-Marital Therapy

Husband and **Wife** tend toward negativity



Post-Marital Therapy

Husband and **Wife** become more positive



!!! Thank You !!! and Questions!

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